201-203-RE - Practice Set #8: Integrals of Trigonometric Functions

Evaluate the following integrals.

(1) $\displaystyle\int_0^{\pi} \left(\sin(2x) - 3e^{-3x}\right)\ dx$

(2) $\displaystyle\int_{\pi/2}^{\pi} \left(\sin(3x) + 6\cos(2x)\right)\ dx$

(3) $\displaystyle\int_0^{\pi/2} \left(4\cos(8x) + 5x^4\right)\ dx$

(4) $\displaystyle\int_{\pi/12}^{\pi/6} \left(9\csc^2(3x) - 36x^3\right)\ dx$

(5) $\displaystyle\int_0^{\pi/16} \left(12\sec^2(4x) - 6e^{2x}\right)\ dx$

(6) $\displaystyle\int_0^{\pi/8} \frac{\sec(2x)\tan(2x)}{4 - \sec(2x)}\ dx$

(7) $\displaystyle\int_{\pi/16}^{\pi/8} \frac{\csc^2(4x)}{\cot(4x) + 1}\ dx$

(8) $\displaystyle\int_0^{\pi} \left(\cos(3x) - x^2\right) e^{\sin(3x) - x^3}\ dx$

(9) $\displaystyle\int_{\pi/8}^{\pi/4} \csc^2(2x)\ e^{1 - \cot(2x)}\ dx$

(10) $\displaystyle\int_0^{\pi/3} (3 - x)\cos(2x)\ dx$

(11) $\displaystyle\int_0^{\pi/6} (2 - 5x)\sin(3x)\ dx$

(12) $\displaystyle\int_0^{\pi/4} \cos(2x)\ (1 + 3\sin(2x))^{1/2}\ dx$

(13) $\displaystyle\int_0^{\pi/12} \frac{\sec^2(3x)}{\sqrt{4 + 3\tan(3x)}}\ dx$

(14) $\displaystyle\int_0^{\pi/12} \frac{\sqrt{1 + 3\sin(2x)}}{\sec(2x)}\ dx$

(15) $\displaystyle\int_0^{\pi/9} \frac{\sec(3x)\tan(3x)}{\sqrt{(3 - \sec(3x))^3}}\ dx$

(16) $\displaystyle\int \frac{\csc(2x)[3x\csc(2x) - 2x^2\sin(2x)]}{6x}\ dx$

(17) $\displaystyle\int \frac{3\sin(2x)[2x^3\csc(2x) + 10x]}{5x}\ dx$

(18) $\displaystyle\int \frac{2[6x\csc(2x) - 5x]}{3x\tan(2x)}\ dx$

(19) $\displaystyle\int \frac{4[6x + 5x\sec(3x)]}{3x\cot(3x)}\ dx$

(20) $\displaystyle\int \sin(3x)[4\cot(3x) - 5\sec(3x)]\ dx$

(21) $\displaystyle\int \cos(3x)[2\csc(3x) + 9\tan(3x)]\ dx$

(22) $\displaystyle\int (8x - 1)\cos(2x)\ dx$

(23) $\displaystyle\int 6(x^2 - 1)\sin(2x)\ dx$

(24) $\displaystyle\int (x^3 + x)\sin(3x)\ dx$

(25) $\displaystyle\int (2x + 3)\csc^2(4x)\ dx$

(26) $\displaystyle\int \frac{5x - 4}{\sec(5x)}\ dx$

(27) $\displaystyle\int 9x^2\csc^2(x^3 + 1)\ dx$

(28) $\displaystyle\int \sec(2x - 3\pi)\ dx$

(29) $\displaystyle\int_0^{\pi/3} \left(9\sin(3x) - 4\sec(x)\tan(x)\right)\ dx$

(30) $\displaystyle\int_{\pi/6}^{\pi/2} \frac{3\sin(x)e^{2x} - \cot(x)}{\sin(x)}\ dx$

(31) $\displaystyle\int_{\pi/12}^{\pi/6} \frac{4\cot(2x) + 3}{\csc(2x)}\ dx$

ANSWERS:

(1) $e^{-3\pi} - 1$

(2) $1/3$

(3) $\dfrac{\pi^5}{32}$

(4) $3 - \dfrac{5\pi^4}{768}$

(5) $6 - 3e^{\pi/8}$

(6) $\dfrac{1}{2}\ln\left(\dfrac{3}{4 - \sqrt{2}}\right)$

1

(7) $\dfrac{1}{4}\ln(2)$

(8) $\dfrac{1}{3}(e^{-\pi^3}-1)$

(9) $\dfrac{1}{2}(e-1)$

(10) $\dfrac{3\sqrt{3}}{4}+\dfrac{3}{8}-\dfrac{\pi\sqrt{3}}{12}$

(11) $\dfrac{1}{9}$

(12) $\dfrac{7}{9}$

(13) $\dfrac{2}{9}(\sqrt{7}-2)$

(14) $\dfrac{5\sqrt{10}}{36}-\dfrac{1}{9}$

(15) $\dfrac{2-\sqrt{2}}{3}$

(16) $-\dfrac{1}{4}\cot(2x)-\dfrac{x^2}{6}+C$

(17) $\dfrac{2}{5}x^3-3\cos(2x)+C$

(18) $-2\csc(2x)-\dfrac{5}{3}\ln|\sin(2x)|+C$

(19) $-\dfrac{8}{3}\ln|\cos(3x)|+\dfrac{20}{9}\sec(3x)+C$

(20) $\dfrac{4}{3}\sin(3x)+\dfrac{5}{3}\ln|\cos(3x)|+C$

(21) $\dfrac{2}{3}\ln|\sin(3x)|-3\cos(3x)+C$

(22) $\dfrac{1}{2}(8x-1)\sin(2x)+2\cos(2x)+C$

(23) $3(1-x^2)\cos(2x)+3x\sin(2x)+\dfrac{3}{2}\cos(2x)+C$

(24) $-\dfrac{1}{3}(x^3+x)\cos(3x)+\dfrac{1}{27}(9x^2+1)\sin(3x)+\dfrac{2}{9}x\cos(3x)+C$

(25) $-\dfrac{1}{4}(2x+3)\cot(4x)+\dfrac{1}{8}\ln|\sin(4x)|+C$

(26) $\dfrac{1}{5}(5x-4)\sin(5x)+\dfrac{1}{5}\cos(5x)+C$

(27) $-3\cot(x^3+1)+C$

(28) $\dfrac{1}{2}\ln|\sec(2x-3\pi)+\tan(2x-3\pi)|+C$

(29) 2

(30) $\dfrac{3}{2}\left(e^\pi-e^{\pi/3}\right)-1$

(31) $\dfrac{7\sqrt{3}-7}{4}$