Math - Calculus II   LIMITS
(let k = nonzero constants)   determinate forms

(1) $\lim\limits_{x\to a} f(x) \cdot g(x) = \infty \cdot k$; (2) $\lim\limits_{x\to a} f(x) \cdot g(x) = 0 \cdot k$; (3) $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\infty}{0}$

(4) $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{k}{0}$; (5) $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{0}{\infty}$; (6) $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{0}{k}$

(7) $\lim\limits_{x\to a} [\, f(x) - g(x)\,] = k-0$; (8) $\lim\limits_{x\to a} [\, f(x) - g(x)\,] = +\infty - 0$

(9) $\lim\limits_{x\to a} [\, f(x) - g(x)\,] = 0-\infty$; (10) $\lim\limits_{x\to a} [\, f(x) - g(x)\,] = k-\infty$; (11) $\lim\limits_{x\to +\infty} \arctan x$

(12) $\lim\limits_{x\to -\infty} \arctan x$; (13) $\lim\limits_{x\to 1} \arctan x$; (14) $\lim\limits_{x\to -1^{+}} \arcsin x$; (15) $\lim\limits_{x\to 0} \arcsin x$

(16) $\lim\limits_{x\to 1^{-}} \arcsin x$; (17) $\lim\limits_{x\to -1^{+}} \arccos x$; (18) $\lim\limits_{x\to +\infty} \operatorname{arcsec} x$; (19) $\lim\limits_{x\to -\infty} \operatorname{arcsec} x$

(20) $\lim\limits_{x\to +\infty} e^{x}$; (21) $\lim\limits_{x\to +\infty} e^{-x}$; (22) $\lim\limits_{x\to -\infty} e^{x}$; (23) $\lim\limits_{x\to -\infty} e^{-x}$; (24) $\lim\limits_{x\to +\infty} e^{-x^{2}}$

(25) $\lim\limits_{x\to -\infty} e^{-x^{2}}$; (26) $\lim\limits_{x\to +\infty} \ln x$; (27) $\lim\limits_{x\to -\infty} \ln x$; (28) $\lim\limits_{x\to e} \ln x$

(29) $\lim\limits_{x\to 0^{+}} \ln x$; (30) $\lim\limits_{x\to 0} \ln x$; (31) $\lim\limits_{x\to \infty} \cos x$; (32) $\lim\limits_{x\to \frac{\pi}{2}^{-}} \tan x$

(33) $\lim\limits_{x\to \frac{\pi}{3}} \tan x$; (34) $\lim\limits_{x\to \frac{\pi}{2}^{-}} \sec x$; (35) $\lim\limits_{x\to \frac{\pi}{2}^{+}} \tan x$

(36) $\lim\limits_{x\to \infty} \dfrac{\sin x}{x}$ or $\lim\limits_{x\to \infty} \dfrac{\cos x}{x}$

Answers:

(1) $\pm\infty$; (2) 0 ; (3) $\pm\infty$; (4) $\pm\infty$; (5) 0 ; (6) 0 ; (7) k ; (8) $+\infty$; (9) $-\infty$; (10) $+\infty$; (11) $\dfrac{\pi}{2}$

(12) $-\dfrac{\pi}{2}$; (13) $\dfrac{\pi}{4}$; (14) $-\dfrac{\pi}{2}$; (15) 0 ; (16) $\dfrac{\pi}{2}$; (17) π ; (18) $\dfrac{\pi}{2}$; (19) $\dfrac{\pi}{2}$; (20) $+\infty$

(21) 0 ; (22) 0 ; (23) $+\infty$; (24) 0 ; (25) 0 ; (26) $+\infty$; (27) undefined ; (28) 1 ; (29) $-\infty$

(30) D.N.E. ; (31) D.N.E. ; (32) $+\infty$; (33) $\sqrt{3}$; (34) ∞ ; (35) $-\infty$; (36) 0

Math - Calculus II   L'HÔPITAL'S RULE

$\dfrac{0}{0}$; $\dfrac{\infty}{\infty}$; $0 \cdot \infty$; $\infty - \infty$; **exponentials**   <u>indeterminate forms</u>

(1) $\displaystyle\lim_{x\to 0} \dfrac{\sin x}{2x}$; (2) $\displaystyle\lim_{x\to 5} \dfrac{\sqrt{x-1}-2}{x^2-25}$; (3) $\displaystyle\lim_{x\to 2} \dfrac{2x^2-5x+2}{5x^2-7x-6}$

(4) $\displaystyle\lim_{x\to 0} \dfrac{\sin x - x}{\tan x - x}$; (5) $\displaystyle\lim_{x\to 0} \dfrac{x+1-e^x}{x^2}$; (6) $\displaystyle\lim_{x\to 0} \dfrac{x-\sin x}{x^3}$

(7) $\displaystyle\lim_{x\to \frac{\pi}{2}^-} \dfrac{2+\sec x}{3\tan x}$; (8) $\displaystyle\lim_{x\to +\infty} \dfrac{x^2}{\ln x}$; (9) $\displaystyle\lim_{x\to 0^+} \dfrac{\ln(\sin x)}{\ln(\sin 2x)}$

(10) $\displaystyle\lim_{x\to 0} \dfrac{e^x - e^{-x} - 2\sin x}{x\sin x}$; (11) $\displaystyle\lim_{x\to +\infty} \dfrac{2x^2+3x+1}{5x^2+x+4}$; (12) $\displaystyle\lim_{x\to +\infty} \dfrac{x\ln x}{x+\ln x}$

(13) $\displaystyle\lim_{x\to +\infty} \dfrac{x^4}{e^x}$; (14) $\displaystyle\lim_{x\to 2^+} \dfrac{\ln(x-1)}{(x-2)^2}$; (15) $\displaystyle\lim_{x\to 0} \dfrac{\arcsin 2x}{\arcsin x}$; (16) $\displaystyle\lim_{x\to 0^+} \dfrac{e^x - \cos x}{x\sin x}$

(17) $\displaystyle\lim_{\theta\to 0} \dfrac{\theta - \sin\theta}{\tan^3\theta}$; (18) $\displaystyle\lim_{x\to \pi} \dfrac{1+\cos 2x}{1-\sin x}$; (19) $\displaystyle\lim_{x\to \frac{1}{2}^-} \dfrac{\ln(1-2x)}{\tan \pi x}$

(20) $\displaystyle\lim_{x\to +\infty} \dfrac{(\ln x)^3}{x}$; (21) $\displaystyle\lim_{x\to +\infty} \dfrac{\dfrac{1}{x^2} - 2\arctan \dfrac{1}{x}}{\dfrac{1}{x}}$; (22) $\displaystyle\lim_{x\to 2}\left(\dfrac{5}{x^2+x-6} - \dfrac{1}{x-2}\right)$

(23) $\displaystyle\lim_{x\to 2} (x-2)\tan \dfrac{\pi x}{4}$; (24) $\displaystyle\lim_{x\to 0} \dfrac{e^{-\frac{1}{x}}}{x}$; (25) $\displaystyle\lim_{x\to +\infty} \dfrac{x}{\sqrt{1+x^2}}$; (26) $\displaystyle\lim_{x\to 0^+} x\ln x$

(27) $\displaystyle\lim_{x\to 0^+} \sin x \ln(\sin x)$; (28) $\displaystyle\lim_{x\to +\infty} x\left(\dfrac{\pi}{2} - \arctan x\right)$

(29) $\displaystyle\lim_{x\to +\infty} x\sin \dfrac{1}{x}$; (30) $\displaystyle\lim_{x\to 0^+} x^{\sin x}$; (31) $\displaystyle\lim_{x\to 0} \left(x^2+e^{2x}\right)^{\frac{1}{x}}$

(32) $\displaystyle\lim_{x\to+\infty}\left(1+\frac{1}{2x}\right)^{x^2}$; (33) $\displaystyle\lim_{x\to 0}(\cos x)^{\frac{1}{x^2}}$; (34) $\displaystyle\lim_{x\to 0^+}(1+x)^{\ln x}$

(35) $\displaystyle\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{5x}$; (36) $\displaystyle\lim_{x\to 0^+}(e^x-1)^x$; (37) $\displaystyle\lim_{x\to+\infty}x^{\frac{1}{x}}$

(38) $\displaystyle\lim_{x\to\frac{\pi}{2}^-}(\tan x)^{\cos x}$; (39) $\displaystyle\lim_{x\to 0^+}(2x+1)^{\cot x}$; (40) $\displaystyle\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\sin x}\right)$

(41) $\displaystyle\lim_{x\to\frac{\pi}{2}^-}(\sec x-\tan x)$; (42) $\displaystyle\lim_{x\to 0^+}\left(\cot x-\frac{1}{x}\right)$

(43) $\displaystyle\lim_{x\to 0^+}\left(\frac{2}{\sin^2 x}-\frac{1}{1-\cos x}\right)$; (44) $\displaystyle\lim_{x\to 1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)$

(45) $\displaystyle\lim_{x\to 2\pi}\left(3-\frac{x}{\pi}\right)^{\csc x}$

Answers:

(1) $\frac{1}{2}$; (2) $\frac{1}{40}$; (3) $\frac{3}{13}$; (4) $-\frac{1}{2}$; (5) $-\frac{1}{2}$; (6) $\frac{1}{6}$; (7) $\frac{1}{3}$; (8) $+\infty$; (9) 1 ; (10) 0

(11) $\frac{2}{5}$; (12) $+\infty$; (13) 0 ; (14) $+\infty$; (15) 2 ; (16) $+\infty$; (17) $\frac{1}{6}$; (18) 2 ; (19) 0 ; (20) 0

(21) -2 ; (22) $-\frac{1}{5}$; (23) $-\frac{4}{\pi}$; (24) 0 ; (25) 1 ; (26) 0 ; (27) 0 ; (28) 1 ; (29) 1

(30) 1 ; (31) e^2 ; (32) $+\infty$; (33) $e^{-1/2}$; (34) 1 ; (35) e^5 ; (36) 1

(37) 1 ; (38) 1 ; (39) e^2 ; (40) 0 ; (41) 0 ; (42) 0 ; (43) $\frac{1}{2}$; (44) $\frac{1}{2}$; (45) $e^{-1/\pi}$