LINES and PLANES

(1) Find the equation of a plane

(a) given one point and $\vec{n}$

(b) given 3 points (noncollinear)

(c) containing 2 vectors $\vec{u} = \left(1, 2, 3\right)$, $\vec{v} = \left(2, -1, 0\right)$

(d) containing 2 intersecting lines

$L_1 : \left(x, y, z\right) = \left(1, -1, 2\right) + t\left(1, 0, 1\right)$

$L_2 : \left(x, y, z\right) = \left(0, 0, 2\right) + s\left(1, -1, 0\right)$

(e) containing 2 parallel lines

$L_1 : \left(x, y, z\right) = \left(1, 2, 9\right) + t\left(0, 1, -4\right)$

$L_2 : \left(x, y, z\right) = \left(2, 0, 5\right) + s\left(0, 1, -4\right)$

(f) containing one point (2, 0, 7) and perpendicular to line $L : \left(x, y, z\right) = \left(1, 2, 9\right) + t\left(0, 1, -4\right)$

(g) containing one point P (3, 0, -1) and a line $L : \left(x, y, z\right) = \left(0, 0, 2\right) + t\left(1, 0, 1\right)$, P is not on L

(h) parallel to another plane $P : 3x + y - z = 20$ and containing a point P (2, 1, 7)

(i) equidistant from 2 points P (2, -1, 3) and Q (1, 1, -1)

(j) perpendicular to 2 planes and containing a point

LINES and PLANES

(2) Find the equation of a line

(a) perpendicular to the plane : $x + 2y - z = 20$ and containing a point (1, 2, 3)

(b) perpendicular to the direction #'s of 2 other non-parallel lines and containing point P

$$P\left(0,1,5\right) ; L_1 : \left(x, y, z\right) = \left(1,1,0\right) + t\left(2,0,-1\right) ; L_2 : \left(x, y, z\right) = \left(2,1,-3\right) + t\left(1,-1,7\right)$$

(c) parallel to 2 non-parallel planes and containing a point P

$$P_1 : x + y + z = 20 ; P_2 : 2x - y + 3z = 10 ; \text{point } P\left(1,2,3\right)$$

(d) containing a point P intersecting another line L and perpendicular to L

$$P\left(1,1,2\right) ; L : \left(x, y, z\right) = \left(2,1,0\right) + t\left(1,1,1\right)$$

$$\text{Answer: } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + t \begin{pmatrix} 4 \\ 1 \\ -5 \end{pmatrix}$$