

FINAL EXAM-201-203-RE  
December 2013

Marks

1. Evaluate each of the following integrals, without the use of integration tables.
- [4] (a)  $\int \frac{(4x-1)(3+x) + 2x \sec^2(2x)}{x} dx$
- [4] (b)  $\int \csc^2 x e^{1-\cot x} dx$
- [4] (c)  $\int_0^{\frac{\pi}{2}} \frac{14 \sin x}{\sqrt{9+7 \cos x}} dx$
- [4] (d)  $\int \frac{5x^2 + 5x - 3}{x^3 + 3x^2} dx$
- [4] (e)  $\int_0^1 2(2x-1) e^{2x} dx$
- [4] (f)  $\int (12x^3 + 15) 2^{x^4+5x} dx$
- [4] (g)  $\int \frac{x}{\sqrt{2x+1}} dx$
- [6] 2. Given the demand function  $p_1(x) = 49 - x^2$  and the supply function  $p_2(x) = 4x + 4$  :
- (a) Find the equilibrium point.
- (b) Sketch and identify the regions representing the consumer and producer surpluses.
- (c) Evaluate the consumer surplus.
- [6] 3. Given the functions  $f(x) = -x^2 + 4x$  and  $g(x) = x^2 - 6$ ,
- (a) Sketch the region R, bounded by the graphs of  $f(x)$  and  $g(x)$ .
- (b) Find the area of the region R.
- [5] 4. Use trapezoidal rule to approximate  $\int_0^{2.5} \sqrt{x^3 + 1} dx$ , using  $n = 5$ . Round your answer to four decimal places.
- [6] 5. Use the table of integrals to find the following integrals. In each case, state the formula number and justify its use.
- (a)  $\int 2x^3 \ln(x^2) dx$
- (b)  $\int \frac{3}{(x+3)^2 \sqrt{x^2 + 6x + 8}} dx$

- [4] 6. Find the cost function  $C(x)$ , given that the marginal cost function  $\frac{dC}{dx} = 0.3x^2 + 6x + 100$  and at 10 units the cost is \$3000.
- [4] 7. Solve the differential equation  $(\cos^2 x) \frac{dy}{dx} = 3y^2$ , with the condition that  $y=1$  when  $x=0$ .
- [5] 8. A software company has the sales volume  $S$  (in millions of dollars) increasing at a rate proportional to the product of the square roots of both the sales volume  $S$  and the time  $t$  (in years). The company started with a sales volume of 36 millions of dollars. A year later, the sales volume reached 64 millions of dollars.
- (a) Find the function  $S(t)$  for the sales volume in dollars at time  $t$  years.
- (b) When will the sales volume reach 484 millions of dollars?
- [6] 9. Evaluate the following limits:
- (a)  $\lim_{x \rightarrow 0} \frac{\ln(x^2)}{\frac{1}{x^4}}$  (b)  $\lim_{x \rightarrow \infty} \frac{3x + e^{6x} + 2}{2e^{6x}}$
- [8] 10. Determine whether the following improper integrals converge or diverge. If the integral converges, find its value.
- (a)  $\int_0^{+\infty} (x-8)^{-\frac{4}{3}} dx$  (b)  $\int_0^4 \frac{(\ln x)^3}{x} dx$
- [3] 11. Consider the sequence  $\left\{ \frac{3}{1}, -\frac{6}{4}, \frac{12}{9}, -\frac{24}{16}, \dots \right\}$
- (a) Give the next two terms of the sequence.
- (b) Find an expression for the  $n$ th term  $a_n$  of the sequence.
- [6] 12. Determine whether the following sequences converge or diverge. If the sequence converges, find the limit.
- (a)  $a_n = \frac{(n+2)!}{(2n^2)n!}$  (b)  $a_n = \frac{3^n}{3n+6}$
- [6] 13. Determine if the following series converge or diverge. If the series converges, find its sum.
- (a)  $\sum_{n=0}^{\infty} \frac{3(2^{n+1})}{5^n}$  (b)  $\sum_{n=1}^{\infty} \frac{3n^2 - n}{n^2 + 1}$
- [4] 14. A deposit of \$25 is made every two weeks into an account for 10 years. If the account pays 2% interest compounded every two weeks, what is the balance in the account after 10 years?
- [3] 15. Use a geometric series to write 3.54 as a ratio of two integers.

## ANSWERS

1. (a)  $2x^2 + 11x - 3 \ln|x| + \tan(2x) + C$ , (b)  $e^{1 - \cot x} + C$ , (c) 4  
(d)  $2 \ln|x| + \frac{1}{x} + 3 \ln|x + 3| + C$ , (e) 2, (f)  $\frac{3 \cdot 2^{x^4 + 5x}}{\ln 2} + C$   
(g)  $\frac{1}{6}(2x + 1)^{\frac{3}{2}} - \frac{1}{2}(2x + 1)^{\frac{1}{2}} + C$
2. Equilibrium Point: (5,24), Consumer's Surplus:  $\frac{250}{3}$ , 3.  $\frac{64}{3} \text{ units}^2$
4. 5.0526 5. (a)  $\frac{1}{2}x^4 \ln(x^2) - \frac{1}{4}x^4 + C$ , (b)  $\frac{3\sqrt{x^2 + 6x + 8}}{x + 3} + C$
6.  $\frac{1}{10}x^3 + 3x^2 + 100x + 1600$ , 7.  $y = \frac{1}{1 - 3 \tan x}$ , 8. (a)  $s = \left(2t^{\frac{3}{2}} + 6\right)^2$
8. (b) 4 years, 9. (a) 0 (b) 1/2 10. (a) Converges to 3/2 (b) Divergent
11. (a)  $a_5 = \frac{48}{25}$ ,  $a_6 = -\frac{96}{36}$ , (b)  $a_n = \frac{3(-2)^{n-1}}{n^2}$ , 12. (a) Converges to 1/2
12. (b) Divergent, 13. (a) Convergent, S= 10, (b) Divergent, 14. \$6,920.26
15.  $\frac{39}{11}$