JOHN
ABBOTT
CEGEP/COLLEGE

$\mathcal{D}$EPARTMENT OF $\mathcal{M}$ATHEMATICS
FINAL EXAMINATION

December 2015

MATHEMATICAL MODELS
201-115-AB

INSTRUCTORS: R.Masters and A. Panassenko
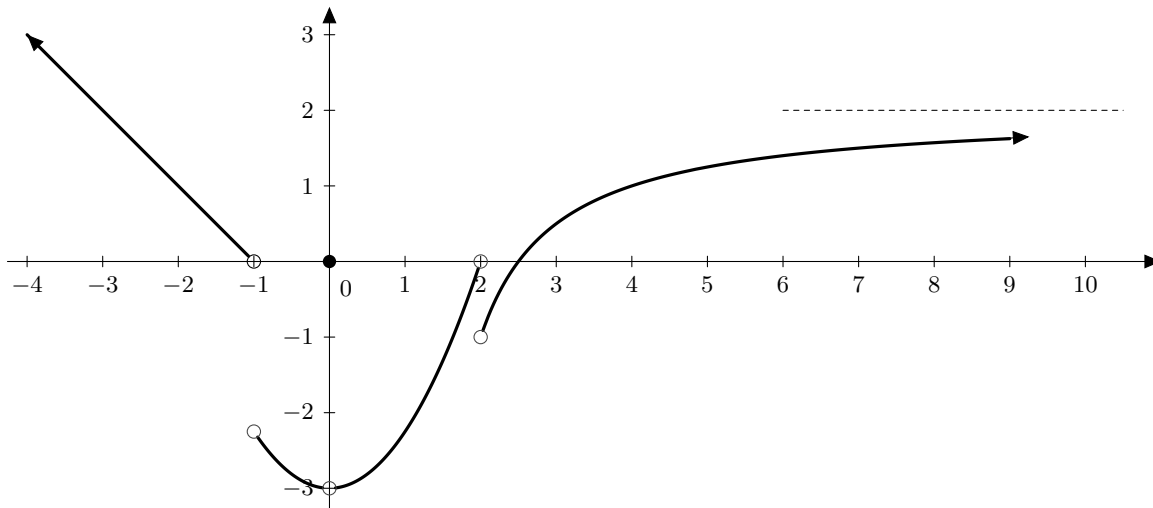
STUDENT NAME: _____

STUDENT NUMBER: _____

INSTRUCTOR: _____

INSTRUCTIONS

1. Do not open this booklet before the examination begins.

2. Check that this booklet contains 5 pages, excluding this cover page and the formula sheet.

3. Write all of your solutions in this booklet and show all supporting work.

4. If the space provided is not sufficient, continue the solution on the opposite page.

(10)   **1.** For the function $f(x)$ given in the diagram below, find each of the following limits. If the limit does not exist, write DNE or $-\infty$ or ∞ where appropriate. If the function is undefined at a point write UND.

    (a) $f(-1) =$ _____

    (b) $f(0) =$ _____

    (c) $\lim\limits_{x \to -1^-} f(x) =$ _____

    (d) $\lim\limits_{x \to -1^+} f(x) =$ _____

    (e) $\lim\limits_{x \to 2} f(x) =$ _____

    (f) $\lim\limits_{x \to 0} f(x) =$ _____

    (g) $\lim\limits_{x \to -\infty} f(x) =$ _____

    (h) $\lim\limits_{x \to \infty} f(x) =$ _____

    (i) List points of discontinuity

(4)   **2.** An ice cream parlor wants to make a giant ice cream cone to advertise their shop. It will consist of an inverted cone, with half a sphere on top. If the radius of the sphere is 1.0m, and the total height of the ice cream 4.5m, find the total volume of the ice cream cone.

3. Perform the indicated operation and express your answer in rectangular form $a + bj$.

(3)   (a) $\dfrac{5j - (3 - j)}{4 - 2j}$

(3)   (b) $(2j^5)(6 - 3j)(4 + 3j)$

(4)   **4.** Solve the following system of equations for z only, using Cramer's Rule

$$
\begin{array}{rcrcrcr}
2x & - & 4y & + & 3z & = & 5 \\
x & & & + & 4z & = & -2 \\
3x & + & y & - & 3z & = & 7
\end{array}
$$

5. Evaluate the following limits if possible.

(3)   (a) $\displaystyle\lim_{x\to\infty} \frac{-2x^2 - 3x}{\sqrt{16x^4 - x}}$

(3)   (b) $\displaystyle\lim_{x\to 5} \frac{\frac{1}{x} - \frac{1}{5}}{5 - x}$

(3)   (c) $\displaystyle\lim_{x\to -3} \frac{x + 3}{x^3 + 27}$

(3)   (d) $\displaystyle\lim_{x\to 0} \frac{\sqrt{x + 9} - 3}{x}$

6. Perform the indicated operations. Express the result in polar form.

(2)   (a) $[3(\cos 60° + j\sin 60°)][4(\cos 30° + j\sin 30°)]$

(4)   (b) $(1.2 + 2.1j)^7$ Hint: use Demoivre's Theorem

(3)   **7.** Consider the function $f(x) = 4\sin\left(\dfrac{\pi}{3}x + \dfrac{\pi}{2}\right) - 1$.

     a) What is the amplitude?

     b) What is the period?

     c) What is the phase shift?

(4)   **8.** 2 hikers start walking from the same spot. The first one walks at 4km/h, bearing 20° north of east. The second one walks at 6.5km/h, bearing 60° north of west. How far apart are they after 2 hours?

(4)   **9.** Find the equation of the tangent line to the curve $f(x) = \sin x + \sin^2 x$ at the point $(0,0)$.

   10. Solve the following for x.

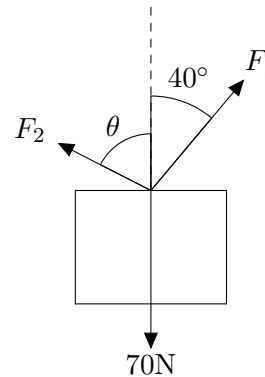(3)   (a) $4^{5-7x} = \dfrac{1}{8^{x-2}}$

(3)   (b) $\log(x - 1) + \log(x + 4) = \log(x + 11)$

(4)   **11.** Solve the following system of equations for each unknown.

$$
\begin{array}{rcrcrcl}
2x &+& 5y &+& 8z &=& 1 \\
3x &+& 9y &+& 17z &=& 9 \\
x &+& 4y &+& 7z &=& 2
\end{array}
$$

(4)   **12.**

A crate has a weight of 70N. You and your friend are pulling on ropes attached to the crate. If you're pulling with a force $F_1 = 64$N, what force F_2 and angle θ must your friend use in order for the crate to be at equilibrium?

$40°$ F_1

F_2 θ

70N

13. Use the diagram below to determine;

$R = 3500\Omega \quad X_c = 2000\Omega$

a b c

$i = 9.00\text{x}10^{-3}\text{A}$
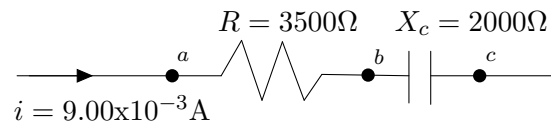
(2)   1. The voltage across the resistor (between points a and b)

(2)   2. The voltage across the capacitor (between points b and c)

(2)   3. The voltage across the combination (between points a and c)

(2)   4. Does the voltage lag or lead the current? If so by what angle.

14. The voltage V induced in an inductor in an electric circuit is given by $V = L \left(\dfrac{d^2 q}{dt^2} \right)$, where the constant L is the inductance (in H).

(3)   (a) Find an expression for the voltage if $q = \sqrt{2t+1} - 1$.

(1)   (b) What is the voltage induced in a 3.00-H inductor when $t = 4s$?

(3)   **15.** At what point(s) is the tangent to the curve $y^2 = 2x^3$ perpendicular to the line $y = \frac{4}{3}x + \frac{1}{3}$?

16. Find y'. (**Do not simplify your answer**)

(3)   (a) $y = \sqrt{\cos \sqrt{x}} + \pi^e - 6x^5 + \log_4 7x$

(3)   (b) $\sin(xy) = x^2 - y^2$

(3)   (c) $y = \ln(2 + 3x^4) + 3^{x \tan x} + \dfrac{1}{x}$

(3)   (d) $y = \dfrac{(x^2 + 1)^4}{(2x + 1)^3}$

17. Solve the following equation for x such that $0 \leqslant x < 2\pi$.

(3)   (a) $2\cos^2(x) + 3\sin(x) - 3 = 0$

(3)   (b) $\tan^2(x) = 2\sec^2(x) - 3$

Mathematical Models 115: Formula Sheet Fall 2015

1. Volume of sphere: $V = \dfrac{4}{3}\pi r^3$

2. Volume of cylinder: $V = \pi r^2 h$

3. Volume of cone: $V = \dfrac{1}{3}\pi r^2 h$

4. Speed = Distance/Time

5. Rectangular: $x + yj$

6. Polar: $r(\cos\theta + j\sin\theta) = r\angle\theta$

7. Exponential: $re^{j\theta}$

8. $x = r\cos\theta \qquad y = r\sin\theta$
   $r = \sqrt{x^2 + y^2}$
   $\tan\theta = \dfrac{y}{x}$

9. DeMoivre's Theorem:
   $[r(\cos\theta + j\sin\theta)]^n = r^n(\cos n\theta + j\sin n\theta)$

10. $V_R = IR$
    $V_C = IX_C$
    $V_L = IX_L$
    $V_{RLC} = IZ$
    $Z = R + j(X_L - X_C)$
    $Z = \sqrt{R^2 + (X_L - X_C)^2}$
    $\theta = \tan^{-1}\dfrac{X_L - X_C}{R}$

11. $\log_b\left(\dfrac{x}{y}\right) = \log_b x - \log_b y$
    $\log_b(xy) = \log_b x + \log_b y$

$\log_b x^p = p\log_b x$
$\log_b b = 1$
$\log_b(1) = 0$
$x = \log_b y \Leftrightarrow b^x = y$

12. $c^2 = a^2 + b^2 - 2ab\cos C$
    $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

13. $(x^3 \pm y^3) = (x \pm y)(x^2 \mp xy + y^2)$

14. $\sin^2 x + \cos^2 x = 1$
    $1 + \tan^2 x = \sec^2 x$
    $1 + \cot^2 x = \csc^2 x$

15. $\sin 2x = 2\sin x\cos x$
    $\cos 2x = \cos^2 x - \sin^x$

16. $\sin^2 x = \dfrac{1 - \cos 2x}{2}$
    $\cos^2 x = \dfrac{1 + \cos 2x}{2}$

17. $\dfrac{d}{dx}(fg) = f'g + fg'$

18. $\dfrac{d}{dx}\left(\dfrac{f}{g}\right) = \dfrac{f'g - fg'}{g^2}$

19. $\dfrac{d}{dx}(x^n) = nx^{n-1}$

20. $\dfrac{d}{dx}(u^n) = nu^{n-1}\dfrac{du}{dx}$

21. $\dfrac{d}{dx}(cf) = cf'$

Answers Fall 2015

1. (a) und

    (b) 0

    (c) 0

    (d) -2.2

    (e) DNE

    (f) -3

    (g) ∞

    (h) 2

    (i) $x = -1; 0; 2$

9. y = x

10. (a) $x = 4/11$

     (b) $x = 3$

2. $\dfrac{11\pi}{6} m^3$

3. $-\dfrac{61}{65}$

11. $x = -21/4; y = -5/4; z = 7/4$

12. $F_2 = 46.3N$

13. (a) $\pi/6; \pi/2; 5\pi/6$

     (b) $\pi/4; 3\pi/4; 5\pi/4; 7\pi/4$

4. (a) $-12 + 66j$

    (b) $-\dfrac{6}{5} + \dfrac{9}{10} j$

14. (a) $V_R = 12v$

     (b) $V_C = 4.5v$

     (c) $V_{RC} = 12.82v$

     (d) $\theta = -20.56°$; Lags

5. (a) $-\dfrac{1}{2}$

    (b) $\dfrac{1}{27}$

    (c) $\dfrac{1}{25}$

    (d) $\dfrac{1}{6}$

15. (a) $y' = -\dfrac{\sin\sqrt{x}}{4\sqrt{x}\cos\sqrt{x}} - 30x^4 + \dfrac{1}{x\ln 4}$

     (b) $y' = \dfrac{2x - y\cos(xy)}{x\cos(xy) + 2y}$

     (c) $y' = \dfrac{12x^3}{2 + 3x^4} + 3^{x\tan x} \cdot \ln 3 \cdot (\tan x + x\sec^2 x) - \dfrac{1}{x^2}$

     (d) $\dfrac{2(x^2 + 1)^3[5x^2 + 4x - 3]}{(2x + 1)^4}$

6. (a) $12[\cos 90° + j\sin 90°]$

    (b) $486[\cos 422.1° + j\sin 422.1°]$

7. (a) 4

    (b) 6

    (c) $-\dfrac{3}{2}$

16. (a) $q''(t) = -\dfrac{1}{(2t + 1)^{3/2}}$

     (b) $V = -\dfrac{1}{9}$

8. 16.40 km