Business Functions

In Business, the following functions are important.
Revenue function $=$ (price per unit) . (quantity of units)
Symbols: $\boxed{R = p \, . \, x}$
Cost function $=$ (average cost per unit) . (quantity of units)
Symbols: $\boxed{C = \overline{C} \, . \, x}$
Profit function $=$ revenue $-$ cost
Symbols: $\boxed{P = R - C}$
Sometimes in a problem some of these functions are given.
Note: Do not confuse p and P.
The price per unit p is also called the demand function p.

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Marginal Functions:
The derivative of a function is called marginal function.

The derivative of the revenue function $R(x)$ is called marginal revenue with notation: $\boxed{R'(x) = \dfrac{dR}{dx}}$

The derivative of the cost function $C(x)$ is called marginal cost with notation: $\boxed{C'(x) = \dfrac{dC}{dx}}$

The derivative of the profit function $P(x)$ is called marginal profit with notation: $\boxed{P'(x) = \dfrac{dP}{dx}}$

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Example 1: Given the price in dollar per unit $p = -3x^2 + 600x$, find:

(a) the marginal revenue at $x = 300$ units. Interpret the result.

   revenue function: $R(x) = p \, . \, x = (-3x^2 + 600x) \, . \, x = -3x^3 + 600x^2$

   marginal revenue: $R'(x) = \dfrac{dR}{dx} = -9x^2 + 1200x$

   marginal revenue at $x = 300 \implies R'(300) = \dfrac{dR}{dx}\bigg|_{x=300} = -9(300)^2 + 1200(300) = -450\,000$

   Interpretation: If production increases from 300 to 301 units, the revenue decreases by $450\,000$ dollars.

(b) the marginal revenue at $x = 100$ units. Interpret the result.

   revenue function: $R(x) = p \, . \, x = (-3x^2 + 600x) \, . \, x = -3x^3 + 600x^2$

   marginal revenue: $R'(x) = \dfrac{dR}{dx} = -9x^2 + 1200x$

   marginal revenue at $x = 100 \implies R'(100) = \dfrac{dR}{dx}\bigg|_{x=100} = -9(100)^2 + 1200(100) = 30\,000$

   Interpretation: If production increases from 100 to 101 units, the revenue increases by $30\,000$ dollars.

Example 2: Given the average cost in dollar per unit $\overline{C} = 357x + 1800$, find:

the marginal cost at $x = 50$ units. Interpret the result.

cost function: $C(x) = \overline{C} \cdot x = (357x + 1800) \cdot x = 357x^2 + 1800x$

marginal cost: $C'(x) = \dfrac{dC}{dx} = 714x + 1800$

marginal cost at $x = 50 \implies C'(50) = \dfrac{dC}{dx}\Big|_{x=50} = 714(50) + 1800 = 37\,500$

Interpretation: If production increases from 50 to 51 units, the cost increases by 37 500 dollars.

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Example 3: Given the revenue function in dollars $R(x) = -3x^3 + 600x^2$ and the cost function in dollars
$\qquad\qquad C(x) = 357x^2 + 1800x$; find:

(a) the marginal profit at $x = 10$ units. Interpret the result.

profit function $=$ revenue $-$ cost

$P(x) = (-3x^3 + 600x^2) - (357x^2 + 1800x) = -3x^3 + 243x^2 - 1800x$

marginal profit: $P'(x) = \dfrac{dP}{dx} = -9x^2 + 486x - 1800$

marginal profit at $x = 10 \implies P'(10) = \dfrac{dP}{dx}\Big|_{x=10} = -9(10)^2 + 486(10) - 1800 = 3300$

Interpretation: If production increases from 10 to 11 units, the profit increases by 3300 dollars.

(b) the marginal profit at $x = 100$ units. Interpret the result.

profit function $=$ revenue $-$ cost

$P(x) = (-3x^3 + 600x^2) - (357x^2 + 1800x) = -3x^3 + 243x^2 - 1800x$

marginal profit: $P'(x) = \dfrac{dP}{dx} = -9x^2 + 486x - 1800$

marginal profit at $x = 100 \implies P'(100) = \dfrac{dP}{dx}\Big|_{x=100} = -9(100)^2 + 486(100) - 1800 = -750\,000$

Interpretation: If production increases from 100 to 101 units, the profit decreases by 750 000 dollars.

Maximum−Minimum Problems: Optimization

To optimize a function means the following:
To maximize the revenue function
To minimize the cost function
To maximize the profit function.
Procedure:
 (a) Define a variable x and build the equation of a function based on the information given in the problem.

 (b) Find the derivative of that function to get the critical number.

 (c) Test the C.N. using the first or second derivative test.

 (d) Answer any question given in the problem.

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Example 4: A manufacturer sells 500 units per week at 31 dollars per unit.
If the price is reduced by one dollar, 20 more units will be sold.

To maximize the revenue, find:

 (a) the selling price
      let x be the number of one dollar reduction
      price in dollars per unit: $31 - x$

 (b) the number of units sold
      number of units sold: $500 + 20x$

 (c) the maximum revenue
      Revenue = (price per unit) . (number of units)
      $R(x) = (31 - x) . (500 + 20x) = -20x^2 + 120x + 15500$

Once the equation of the revenue function is found, use the first derivative to find C.N.,
test the C.N. using the first or second derivative and then answer the following questions:
(1) find the selling price to maximize the revenue;
(2) find the number of units sold to maximize the revenue;
(3) find the maximum revenue.

$R'(x) = -40x + 120 \implies R'(x) = 0 \implies -40x + 120 = 0 \implies x = 3$

Test the critical number $x = 3$ with the second derivative: $R''(x) = -40 < 0$, relative maximum

(1) the selling price to maximize the revenue is $31 - 3 = 28$ dollars per unit
(2) the number of units sold to maximize the revenue is $500 + 20(3) = 560$ units
(3) the maximum revenue is $R(3) = (28) . (560) = \$15\,680$

Example 5: A rectangular tennis court of 1800 square meters is to be fenced with 2 types of materials.
The shorter sides are made with fence material costing \$100 per meter and the other sides with fence material costing
\$50 per meter. Find the dimensions of the court to minimize cost.

Let x be the shorter side and y be the other side in meter.

Total cost of the fence: $C = 2x(100) + 2y(50) = 200x + 100y$

Area: $1800 = x \cdot y \implies y = \dfrac{1800}{x}$

replace y in the equation of C :

$$C(x) = 200x + 100\dfrac{1800}{x} = 200x + \dfrac{180\,000}{x}$$

$$C'(x) = 200 - \dfrac{180\,000}{x^2} \implies C'(x) = 0 \implies 200 - \dfrac{180\,000}{x^2} = 0 \implies x = 30$$

Test the critical number $x = 30$ with the second derivative: $C''(x) = \dfrac{360\,000}{x^3} \implies C''(30) > 0$, relative minimum

To minimize the cost, the dimensions of the court should be $x = 30$ meters by $y = \dfrac{1800}{30} = 60$ meters.

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Example 6: A company has established that the revenue function in dollars is $R(x) = 2x^3 + 40x^2 + 8x$ and the cost
function in dollars is $C(x) = 3x^3 + 19x^2 + 80x - 800$.
Find the price per unit to maximize the profit.

To maximize the profit, need the profit function $P = R - C$

$$P(x) = (2x^3 + 40x^2 + 8x) - (3x^3 + 19x^2 + 80x - 800) = -x^3 + 21x^2 - 72x + 800$$

Once the equation of the profit function is found, use the first derivative to find C.N.,
test the C.N. using the first or second derivative and then answer the question:

$$P'(x) = -3x^2 + 42x - 72 \implies P'(x) = 0 \implies -3x^2 + 42x - 72 = 0 \implies -3(x - 2)(x - 12) = 0$$

the critical numbers are: $x = 2$; $x = 12$

use second derivative test: $f''(x) = -6x + 42$

$x = 2 \implies f''(2) > 0 \implies$ relative minimum ; $x = 12 \implies f''(12) < 0 \implies$ relative maximum

To maximize the profit, use $x = 12$. To find the price per unit:

$$p = \dfrac{R}{x} = \dfrac{2x^3 + 40x^2 + 8x}{x} = 2x^2 + 40x + 8$$

at $x = 12 \implies p(12) = 2(12)^2 + 40(12) + 8 = \$776$ per unit.

Price Elasticity of Demand

Definition: price elasticity of demand is a ratio of: $\boxed{\dfrac{\% \text{ change in quantity}}{\% \text{ change in price}}}$

Notation and formula of price elasticity of demand:

Symbol of price elasticity of demand is the Greek letter η pronounced "ETA"

Formula of price elasticity of demand: $\boxed{\eta = \dfrac{p}{x \cdot p'}}$

The proof is given in class.

where p is the demand function (price per unit) ; x is the demanded quantity (number of units) and p' is the derivative of p.

Price Elasticity of Demand Interpretation:

if $\;|\eta| > 1 \iff \eta < -1\;$ or $\;\eta > 1 \longrightarrow$ demand is elastic

if $\;|\eta| < 1 \iff -1 < \eta < 1 \longrightarrow$ demand is inelastic

if $\;|\eta| = 1 \iff \eta = \pm 1 \longrightarrow$ demand has unit elasticity

Elastic demand $\longrightarrow$ decrease price implies increase revenue.
Inelastic demand $\longrightarrow$ decrease price implies decrease revenue.
Unit elasticity of demand $\longrightarrow$ decrease price implies unchanged revenue.

Example 7: The demand of a product is $p = 25 - x^2$ where x is the demanded quantity. Find:
(a) the price elasticity of demand.

Use formula $\eta = \dfrac{p}{x \cdot p'}$; need to replace $p = 25 - x^2$ and $p' = -2x$

$$\eta = \frac{25 - x^2}{x \cdot (-2x)} = \frac{x^2 - 25}{2x^2}$$

(b) the intervals of elasticity

Two equations to solve for x :

$$\frac{x^2 - 25}{2x^2} = 1 \implies x^2 - 25 = 2x^2 \implies -25 = x^2 \implies \text{no solution}$$

$$\frac{x^2 - 25}{2x^2} = -1 \implies x^2 - 25 = -2x^2 \implies 3x^2 - 25 = 0 \implies x^2 = \tfrac{25}{3} \implies x \approx \pm 2.89$$

Next page, testing on the number line with $x = 2.89$ and $x = 0$ to $x = 5$
since p cannot be negative and cannot exceed $x = 5$.

Example 7 (continues):
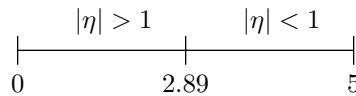
Use $\quad \eta = \dfrac{x^2 - 25}{2x^2}$

choose in first interval $0 < x < 2.89 \longrightarrow x = 1 \ :\quad \eta(1) = \dfrac{(1)^2 - 25}{2(1)^2} = -12$

$|\eta| > 1 \longrightarrow$ demand is elastic at $0 < x < 2.89$

choose in second interval $2.89 < x < 5 \longrightarrow x = 3 \ :\quad \eta(3) = \dfrac{(3)^2 - 25}{2(3)^2} \approx -0.89$

$|\eta| < 1 \longrightarrow$ demand is inelastic at $2.89 < x < 5$

Elasticity Testing:

$$|\eta| > 1 \qquad\qquad |\eta| < 1$$
$$\vdash\!+\!\dashv$$
$$0 \qquad\qquad 2.89 \qquad\qquad 5$$

Graph of the demand of a product $p = 25 - x^2$ and the revenue $R(x) = p \cdot x = (25 - x^2) \cdot x = 25x - x^3$
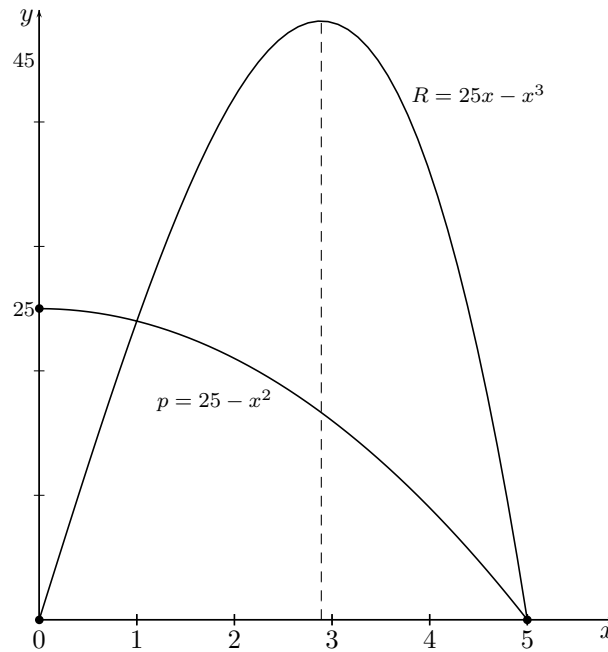
The graph shows in the interval

$0 < x < 2.89$, if the price decreases,

revenue increases implies demand is elastic.

In the interval $2.89 < x < 5$,

if the price decreases, revenue decreases

implies demand is inelastic.

At $x = 2.89$, the revenue is maximum.

This graph is only visual!
(not part of the solution)

Example 8: The demand of a product is $p = \dfrac{10}{x+2}$ where x is the demanded quantity. Find:

(a) the price elasticity of demand.

Use formula $\eta = \dfrac{p}{x \cdot p'}$; need to replace $p = \dfrac{10}{x+2}$ and $p' = -\dfrac{10}{(x+2)^2}$

$$\eta = \dfrac{\dfrac{10}{x+2}}{x \cdot \left(-\dfrac{10}{(x+2)^2}\right)} = \dfrac{10}{x+2} \cdot \dfrac{(x+2)^2}{-10x} = -\dfrac{x+2}{x}$$

(b) If at $x = 8$, the price decreases by 2%, what is the percentage change in the quantity?

Use the definition of price elasticity of demand:

$$\eta = \dfrac{\% \text{ change in quantity}}{\% \text{ change in price}}$$

at $x = 8$; $p = \dfrac{10}{8+2} = 1$ and $\eta(8) = -\dfrac{8+2}{8} = -1.25$

replace $\eta(8) = -1.25$ and $\%$ change in price $= -2\%$, then

$$-1.25 = \dfrac{\% \text{ change in quantity}}{-2\%} \implies \% \text{ change in quantity} = (-1.25)(-2\%) = +2.5\%$$

(c) the intervals of elasticity

Two equations to solve for x :

$$-\dfrac{x+2}{x} = 1 \implies -x - 2 = x \implies -2 = 2x \implies x = -1$$

$$-\dfrac{x+2}{x} = -1 \implies -x - 2 = -x \implies -2 = 0 \implies \text{no solution}$$

Both answers are not acceptable for testing since $x > 0$ (only positive quantity)

Next page, testing on the number line with $x = 0$ to $x = +\infty$ since p cannot be negative.
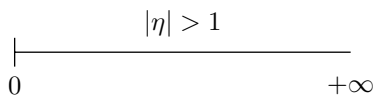
Example 8 (continues):

Use $\quad \eta = -\dfrac{x+2}{x}$

choose in the interval $0 < x < +\infty \longrightarrow x = 8$: $\quad \eta(8) = -\dfrac{(8)+2}{(8)} = -1.25$

$|\eta| > 1 \longrightarrow$ demand is elastic at $0 < x < +\infty$; the demand is always elastic

Elasticity Testing:

$$|\eta| > 1$$

$\vdash\!\!\!\!\rule{6cm}{0.4pt}$

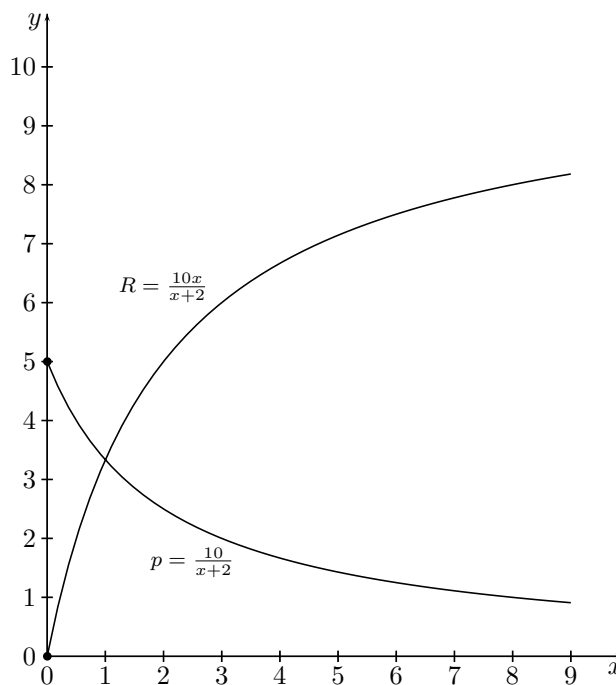$0 \qquad\qquad\qquad\qquad\qquad\qquad +\infty$

Graph of the demand of a product $p = \dfrac{10}{x+2}$ and the revenue $R(x) = p \, . \, x = \dfrac{10}{x+2} \, . \, x = \dfrac{10x}{x+2}$

The graph shows that the

demand is always elastic.

If the price decreases,

the revenue increases.

This graph is only visual!
(not part of the solution)

Applications of the Derivative: Maximum and Minimum Problems

EXAMPLE 1:

A total of x earthmovers will be sold if the price, in thousands of dollars is given by $p = 32 - \dfrac{x}{8}$.

Find:

(a) an expression for the total revenue $R(x)$. $\implies R(x) = p \cdot x = x\left(32 - \dfrac{x}{8}\right) = 32x - \dfrac{x^2}{8}$

(b) the value of x that leads to maximum revenue. $\implies x = 128$

(c) the maximum revenue. $\implies R(128) = 2048$ thousands of dollars

EXAMPLE 2:

An investor has built a series of self-storage units near a group of apartment houses. He must decide on the monthly rental. From past experience, he feels that 200 units will be rented for \$15 per month with 5 additional rentals for each 25 cents reduction in the rental price. Let x be the number of 25 cents reductions in the price and find:

(a) an expression for the number of units rented. $\implies 200 + 5x$

(b) an expression for the price per unit. $\implies 15 - 0.25x$

(c) an expression for the total revenue. $\implies R = (200 + 5x)(15 - 0.25x) = -1.25x^2 + 25x + 3000$

(d) the value of x leading to maximum revenue. $\implies x = 10$

(e) the maximum revenue. $\implies R(10) = \$3125$

EXAMPLE 3:

A diesel generator burns fuel at the rate of $G(x) = \dfrac{1}{12}\left(\dfrac{338}{x} + 2x\right)$ litres per hour when producing x thousand kilowatt hours of electricity. Suppose that fuel costs 60 cents a litre and find the value of x that leads to minimum total cost if the generator is operated for 32 hours.
Find the minimum cost.

$$C(x) = (32)(0.60)\dfrac{1}{12}\left(\dfrac{338}{x} + 2x\right) = 1.6\left(\dfrac{338}{x} + 2x\right)$$

critical number $x = 13$ thousands of KWH $\implies C(13) = \$83.20$

Applications of the Derivative: Maximum and Minimum Problems

1. A dress manufacturer has found that the number of dresses sold per month (x) , is related to the price in dollars per dress (p) by the equation $p = 500 - \frac{x^2}{300}$.
   Determine whether total revenue is increasing or decreasing when $x = 100, 200, 300$.

2. A company manufacturing bathing suits sells the suits at $24 each.
   The dollar cost of producing x suits is $C(x) = 150 + \frac{39}{10} x + \frac{3}{1000} x^2$.
   Express profit as a function of x and determine the number of bathing suits the company should produce to achieve maximum profit.

3. Boxed greeting cards cost the distributor 60 cents/box. He can sell 100 boxes at $1/box.
   For each cent the price is lowered he can increase the number of boxes sold by 5.

   (a) If x is the number of boxes sold, show that the total revenue in cents is $R(x) = \frac{1}{5} \left(600x - x^2\right)$.

   (b) Define the profit function, $P(x)$.

   (c) How many boxes should he sell to maximize profit ?
       What would the price be then ?   What would the profit be in $?

4. A restaurant chain is planning a new dining room. Estimates of monthly profit per chair, indicate that for 100 chairs the profit/chair would be $24.
   If seating capacity is over 100 chairs, the monthly profit/chair decreases by 10 cents per chair added.
   What seating capacity will maximize profit ?

5. A Florida orange grower finds that the average yield/tree is 400 oranges if no more than 16 trees are planted in each plot. For each additional tree per plot the yield of each tree is decreased by 20 oranges.
   How many trees should be planted in each plot to maximize yield?

6. An all news radio station has made a study of the listening habits of local residents between 5 p.m. and midnight . The results indicate that x hours after 5 p.m. on a typical week night: $-\frac{x^3}{4} + \frac{27}{8} x^2 - \frac{27}{2} x + 30$ percent of the population is tuned into the station.

   (a) At what time between 5 p.m. and midnight are there most listeners ?

   (b) At what time between 5 p.m. and midnight are there fewest listeners ?

7. Plans for a drugstore require 14,400 square feet of floor space. The floor will be rectangular with 3 brick walls and an all glass front.

   (a) If glass costs 1.88 times as much as brick (brick costs $50 a linear foot), which dimensions of floor would minimize cost of materials ?

   (b) If heat loss is 7 times as great through glass as through brick (heat loss through brick is 50 kcal a linear foot), which dimensions of floor would minimize heat loss ?

8. The daily production cost for a factory to manufacture x deluxe contour chairs is given to be $ $\left(500 + 14x + \frac{x^2}{2}\right)$.
   The price function is $ $\left(150 - \frac{3}{2} x\right)$.

   (a) Write the equation of the revenue function, $R(x)$.

   (b) Write the equation of the profit function, $P(x)$.

   (c) Evaluate the marginal cost for $x = 10, \ x = 30, \ x = 50$.
       Evaluate the marginal profit for $x = 10, \ x = 30, \ x = 50$.

   (d) How many chairs should be produced daily to maximize the profit ?

Applications of the Derivative: Maximum and Minimum Problems

9. $\$\left(\dfrac{x^2}{3} + 34x + 40\right)$ is the total cost of production per day for x Cassette players and they are sold for $\$\left(62 - \dfrac{x}{4}\right)$ per Cassette player. What daily production will produce a maximum profit ?

10. A stereo manufacturer determines that in order to sell x units of a new stereo, its price per unit must be $p = 1000 - x$. It also determines that the total cost of producing x units is given by $C(x) = 3000 - 20x$.

  (a) Find the total revenue equation, $R(x)$.

  (b) Find the total profit equation, $P(x)$.

  (c) How many units must the company produce and sell to maximize profit ?

  (d) What is the maximum profit?

  (e) What price per unit must be charged to make maximum profit?

11. From a thin piece of cardboard, 8 in. by 8 in., square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume ? What is the maximum volume ?

12. Of all numbers whose sum is 50, find the two which have maximum product.
Can there be a minimum product ? Explain.

13. Suppose you are the owner of a 95 unit motel. All units are occupied when you charge $45 a day per unit. For every increase of x dollars in the daily rate, there are x units vacant. Each occupied room costs $20 a day to service and maintain. What should you charge per unit to maximize profit ?

14. A truck burns fuel at the rate of $G(x) = \dfrac{1}{30}\left(\dfrac{8100}{x} + x\right)$ litres per km when travelling x km per hour on a straight level road. If fuel costs 60 cents per litre, find the speed that will produce the minimum total cost for a 1000-km trip.

15. A railroad company offers the following discount rates for chartered trips of at least 400 passengers; the fare is $40 per person for exactly 400 passengers, while the fare per person decreases by five cent for each passenger over 400. Find the number of passengers which will yield the maximum revenue.
What is the maximum revenue?

16. The National Forest Service finds that 16 trees planted on an acre of land will grow approximately 3 feet per year. For each additional tree planted, the growth will be reduced by 0.5 inch.
Find the number of trees per acre that will yield the largest amount of timber (wood).

17. A group of students arrange a chartered flight to Paris. The charge per student is $800 if 100 students go on the flight. If more than 100 participate, the charge per student is reduced by an amount equal to $4 times the number of students above 100.

  (a) Find the total revenue equation, R, as a function of x (the number of students above 100).

  (b) Find the number of students that will provide maximum revenue.

  (c) What is the charge per student when maximum revenue is obtained.

18. A school wants to enclose behind a wall of the building 2 adjacent equal rectangular courtyards of total area 19200 square feet with a fence that costs $50 per foot. Find the dimensions of each courtyard to minimize the cost of the fence. Find the minimum cost.

19. A sports complex wants to enclose 3 adjacent equal tennis courts with a fence that costs $20 per linear meter. The budget of the fence is $12 000 ; find the dimensions of each tennis court to maximize the area. Find the maximum area.

20. A farmer wants to enclose 2 adjacent equal rectangular fields of total area 12000 square meters. The exterior fence costs \$50 per meter and the interior fence costs \$20 per meter. Find the dimensions of each field to minimize the cost of the fence. Find the minimum cost.

Answers:

(1) $R'(100) > 0$; $R'(200) > 0$; $R'(300) < 0$

(2) $P(x) = -\dfrac{3}{1000}x^2 + \dfrac{201}{10}x - 150$; to maximize profit: 3350 Bathing suits.

(3 b) $P(x) = \dfrac{3}{50}x - \dfrac{x^2}{500}$ dollars ; (3 c) 150 boxes ; Price/unit: 90 cents ; Max $P(150) = \$45$

(4) 170 chairs   (5) 18 trees/plot

(6 a) Maximum: Most listeners at 5:00 P.M.   (6 b) Minimum: Fewest listeners at 8:00 P.M.

(7 a) 100 ft by 144 ft ; (7 b) 60 ft by 240 ft

(8 a) $R(x) = 150x - \dfrac{3}{2}x^2$ (8 b) $P(x) = -2x^2 + 136x - 500$

(8 c) marginal cost: $C'(x) = 14 + x \implies C'(10) = 24$, $C'(30) = 44$, $C'(50) = 64$ \$/chair
marginal profit: $P'(x) = -4x + 136 \implies P'(10) = 96$, $P'(30) = 16$, $P'(50) = -64$ \$/chair
(8 d) 34 chairs

(9) 24 cassette players

(10 a) $R(x) = 1000x - x^2$ (10 b) $P(x) = -x^2 + 1020x - 3000$ (10 c) 510 units
(10 d) Maximum profit: $P(510) = \$257,100$ (10 e) Price per unit: $p = \$490$

(11) dimensions of the box are 16/3 inches by 16/3 inches by 4/3 inches with maximum volume of the box is about 37.9 cubic inches

(12) the 2 numbers are 25 and 25   (13) \$80   (14) speed is 90 km/h

(15) 600 passengers ; maximum revenue is \$18000   (16) 44 trees/acre

(17 a) $R(x) = 80000 + 400x - 4x^2$ (17 b) 150 students   (17 c) Charge/student: \$600

(18) dimensions are 80 by 120 ft each courtyard ; minimum cost is \$24000

(19) dimensions are 75 by 50 m each tennis court ; maximum area is 11250 square meters

(20) dimensions of each field are 100 by 60 meters ; minimum cost is \$24000

Price Elasticity of Demand

1. If the price of a product increases by 6% and the demanded quantity decrease by 2%; find the price elasticity of demand.

2. Given the demand function $p = \dfrac{1500}{x + 10}$

    (a) Find the price elasticity of demand
    (b) Evaluate the elasticity of demand at $x = 20$
    (c) Find the intervals of elasticity
    (d) Describe the elasticity and interpret the result in (b)
    (e) If the price of the product decreases by 2% at the level of $x = 20$, what is the percent change in quantity?

3. Given the demand function $p = 42 + x - x^2$

    (a) Find the price elasticity of demand
    (b) Evaluate the elasticity of demand at $p = 22$
    (c) Find the intervals of elasticity
    (d) Describe the elasticity and interpret the result in (b)

4. Given the demand function $p = \dfrac{2500 - 10x}{x + 9}$

    (a) Find the price elasticity of demand
    (b) Evaluate the elasticity of demand at $x = 40$
    (c) Find the intervals of elasticity
    (d) Describe the elasticity and interpret the result in (b)

5. Given the demand function $p = 100 - 4x$

    (a) Find the price elasticity of demand
    (b) Evaluate the elasticity of demand at $p = 20$
    (c) Find the intervals of elasticity
    (d) Describe the elasticity and interpret the result in (b)

6. Given the demand function $p = \sqrt{4 - x^2}$

    (a) Find the price elasticity of demand
    (b) Evaluate the elasticity of demand at $x = 1$
    (c) Find the intervals of elasticity
    (d) Describe the elasticity and interpret the result in (b)

Price Elasticity of Demand

7. Given the demand function $p = \dfrac{50}{\sqrt{100 + x}}$

   (a) Find the price elasticity of demand

   (b) Evaluate the elasticity of demand at $x = 25$

   (c) Describe the elasticity and interpret the result in (b)

   (d) If the price of the product decreases by 2% at the level of $x = 25$, what is the percent change in quantity?

8. Given the demand function $p = \dfrac{5000}{x + 10} - 125$

   (a) Find the price elasticity of demand

   (b) Evaluate the elasticity of demand at $x = 15$

   (c) Find the intervals of elasticity

   (d) Describe the elasticity and interpret the result in (b)

9. The demand function of a product is given by $p = \dfrac{50000 - 20x}{x + 9}$ where p is the price per unit when x units are demanded.

   (a) Determine the price elasticity of demand when $x = 1000$

   (b) Is the demand elastic, inelastic or has unit elasticity at $x = 1000$?

   (c) At $x = 1000$ the price of the product decreases by 3.5%, what is the approximate percentage change in quantity?

10. The demand function of a product is given by $p = \dfrac{1000}{(x + 1)^2}$ where p is the price per unit when x units are demanded.

    (a) Determine the price elasticity of demand when $p = 2.50$

    (b) Is the demand elastic, inelastic or has unit elasticity at $p = 2.50$?

    (c) At $p = 2.50$ the price of the product decreases by 4%, what is the approximate percentage change in quantity?

    (d) Will the total revenue increase, decrease or remain unchanged at $p = 2.50$?

11. The demand function of a product is given by $p = \dfrac{50000 - 1000x}{x + 60}$ where p is the price per unit when x units are demanded.

    (a) Determine the price elasticity of demand when $p = 100$

    (b) Is the demand elastic, inelastic or has unit elasticity at $p = 100$?

    (c) At $p = 100$ the price of the product decreases by 2%, what is the approximate percentage change in quantity?

    (d) Will the total revenue increase, decrease or remain unchanged at $p = 100$?

12. The demand function of a product is given by $p = -x + 40 + \dfrac{6000}{x}$ where p is the price per unit when x units are demanded.

    (a) Determine the price elasticity of demand when $p = 320$

    (b) Is the demand elastic, inelastic or has unit elasticity at $p = 320$?

    (c) Will the total revenue increase, decrease or remain unchanged at $p = 320$?

    (d) At $p = 320$ the price of the product decreases by 2.5%, what is the approximate percentage change in quantity?

    (e) Find the intervals of elasticity.

Price Elasticity of Demand

Answers:

(1) $-\frac{1}{3}$, (2 a) $\eta = -\frac{x+10}{x}$; (2 b) -1.5 ; (2 c) elastic at $x > 0$

(2 d) demand is elastic ; if the price decreases by 10% , the quantity increases by 15% and revenue increases.

(2 e) the quantity will increase by 3% ; (3 a) $\eta = \dfrac{42 + x - x^2}{x - 2x^2}$; (3 b) -0.49

(3 c) elastic at $0 < x < 4.09$; inelastic at $4.09 < x < 7$; has unit elasticity at $x = 4.09$

(3 d) demand is inelastic ; if the price decreases by 10% ,
    the quantity increases by 4.9% and revenue decreases.

(4 a) $\eta = \dfrac{(x + 9)(x - 250)}{259x}$ (4 b) -0.99

(4 c) elastic at $0 < x < 39.28$; inelastic at $39.28 < x < 250$; has unit elasticity at $x = 39.28$

(4 d) demand is inelastic ; if the price decreases by 10% ,
    the quantity increases by 9.9% and revenue decreases.

(5 a) $\eta = \dfrac{4x - 100}{4x}$ (5 b) -0.25

(5 c) elastic at $0 < x < 12.5$; inelastic at $12.5 < x < 25$; has unit elasticity at $x = 12.5$

(5 d) demand is inelastic ; if the price decreases by 10% ,
    the quantity increases by 2.5% and revenue decreases.

(6 a) $\eta = \dfrac{x^2 - 4}{x^2}$ (6 b) -3

(6 c) elastic at $0 < x < 1.41$; inelastic at $1.41 < x < 2$; has unit elasticity at $x = 1.41$

(6 d) demand is elastic ; if the price decreases by 1% , the quantity increases by 3% and revenue increases.

(7 a) $\eta = -\dfrac{2(100 + x)}{x}$ (7 b) -10

(7 c) demand is elastic ; if the price decreases by 1% ,
    the quantity increases by 10% and revenue increases.

(7 d) the quantity will increase by 20%

(8 a) $\eta = \dfrac{(x + 10)(x - 30)}{40x}$ (8 b) -0.625

(8 c) elastic at $0 < x < 10$; inelastic at $10 < x < 30$; has unit elasticity at $x = 10$

(8 d) demand is inelastic ; if the price decreases by 10% ,
    the quantity increases by 6.25% and revenue decreases.

(9 a) -0.6 ; (9 b) inelastic ; (9 c) +2.1%

(10 a) -0.5263 ; (10 b) inelastic ; (10 c) +2.11% ; (10 d) revenue will decrease.

(11 a) -0.23 ; (11 b) inelastic ; (11 c) +0.46% ; (11 d) revenue will decrease.

(12 a) -1 ; (12 b) demand has unit elasticity ; (12 c) revenue remain constant ; (12 d) +2.5%

(12 e) elastic at $0 < x < 20$; inelastic at $20 < x < 100$; has unit elasticity at $x = 20$