Derivative formulas of Exponential Function

The derivative formula of the exponential function is $y = f(x) = a^x \implies y' = f'(x) = \dfrac{dy}{dx} = a^x \ln(a)$

Composite exponential function where u is a function of x: Chain Rule for exponential function

$y = f(x) = a^u \implies y' = f'(x) = \dfrac{dy}{dx} = a^u \ln(a) \,.\, u'$

base 10 : $y = 10^x \implies y' = 10^x \ln(10)$

base e : $y = e^x \implies y' = e^x$

Example: If $y = e^{-x^3} \implies y' = e^{-x^3} \left(-x^3\right)' = -3x^2 e^{-x^3}$

Example 1 of Exponential Function:

Given $f(x) = e^{-2x}$, find the equation of the tangent line at $x = 0$.

The derivative

$f'(x) = e^{-2x} \left(-2x\right)' = -2\, e^{-2x}$

point at $x = 0$, $y = 0$

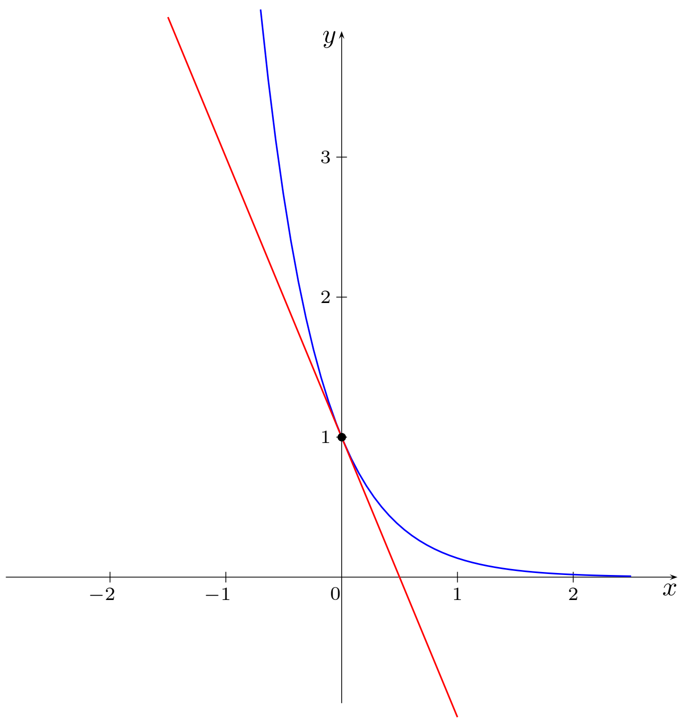
slope: $m = f(0) = -2$

equation of tangent line:

$y - 1 = -2(x - 0) \implies y = -2x + 1$

The graph shows the curve of $f(x)$ (blue curve)
and the tangent line at $x = 0$ (red line)

Note: The graph is not part of the solution.
It is just visual !



Example 2 of Exponential Function:

Given $f(x) = (2x-1)\,e^x$, find the equation of the tangent line at $x = 0$.

The Product Rule: $(u.v)' = u'v + v'u$

$u = 2x - 1 \rightarrow u' = 2$; $v = e^x \rightarrow v' = e^x$

$f'(x) = 2\,e^x + e^x\,(2x-1) = (2x+1)\,e^x$

point at $x = 0$, $y = -1$

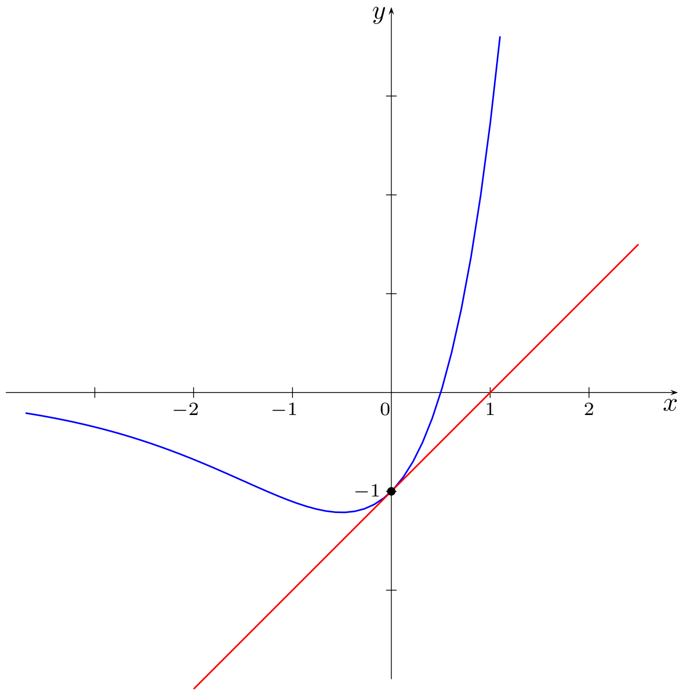
slope: $m = f(0) = 1$

equation of tangent line:

$y - (-1) = 1(x - 0) \implies y = x - 1$

The graph shows the curve of $f(x)$ (blue curve)
and the tangent line at $x = 0$ (red line)

Note: The graph is not part of the solution.
It is just visual !



Example 3 of Exponential Function:

Given $f(x) = \dfrac{x-2}{e^x}$, find the equation of the tangent line at $x = 0$.

The Quotient Rule: $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - v'u}{v^2}$

$u = x - 2 \rightarrow u' = 1$; $v = e^x \rightarrow v' = e^x$

$f'(x) = \dfrac{(1)\,e^x - e^x\,(x-2)}{(e^x)^2} = \dfrac{(3-x)\,e^x}{(e^x)^2} = \dfrac{3-x}{e^x}$

point at $x = 0$, $y = -2$

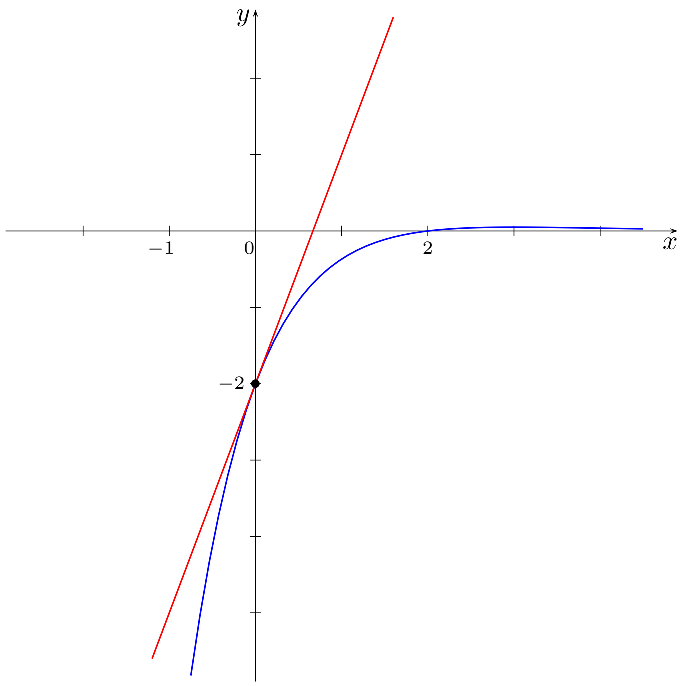
slope: $m = f(0) = 3$

equation of tangent line:

$y - (-2) = 3(x - 0) \implies y = 3x - 2$

The graph shows the curve of $f(x)$ (blue curve)
and the tangent line at $x = 0$ (red line)

Note: The graph is not part of the solution.
It is just visual !



Example 4 of Exponential Function:

Given $f(x) = e^{1-x^2}$, find the equation of the tangent line at $x = 1$.

The Chain Rule: $\Big(u(v)\Big)' = u'(v)\,.\,v'$

$u(v) = e^v \rightarrow u'(v) = e^v\,.\,v'$

$v = 1 - x^2 \rightarrow v' = -2x$

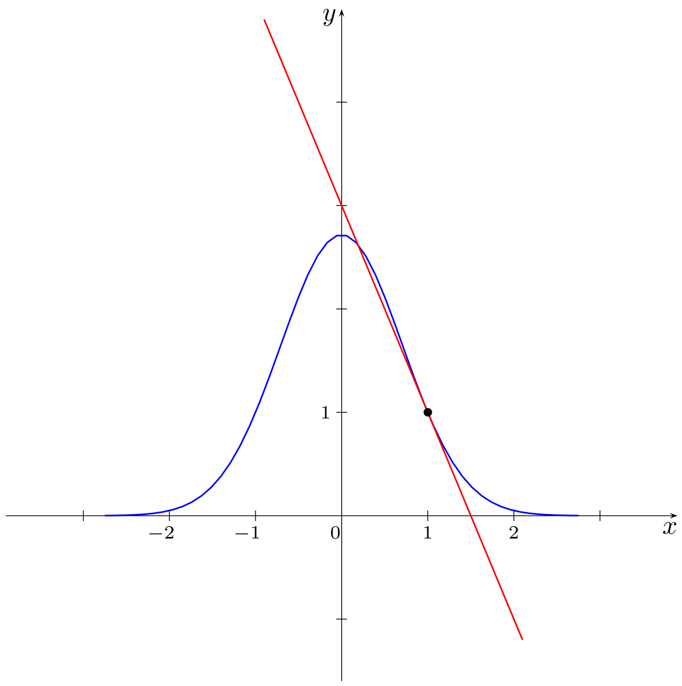
$f'(x) = e^{1-x^2}\,.\,(-2x) = -2x\,e^{1-x^2}$

point at $x = 1$, $y = 1$

slope: $m = f(1) = -2$

equation of tangent line:

$y - 1 = -2(x-1) \implies y = -2x + 3$



The graph shows the curve of $f(x)$ (blue curve) and the tangent line at $x = 1$ (red line)

<u>Note</u>: The graph is not part of the solution.
It is just visual !

Derivative formulas of Logarithmic Function

The derivative formula of the logarithmic function is $y = f(x) = \log_a(x) \implies y' = f'(x) = \dfrac{dy}{dx} = \dfrac{1}{x\,\ln(a)}$

Composite logarithmic function where u is a function of x: Chain Rule for logarithmic function

$$y = f(x) = \log_a(u) \implies y' = f'(x) = \frac{dy}{dx} = \frac{1}{u\,\ln(a)}\,.\,u'$$

base 10 : $y = \log(x) \implies y' = \dfrac{1}{x\,\ln(10)}$

base e : $y = \log_e(x) = \ln(x) \implies y' = \dfrac{1}{x}$

Example: If $y = \ln(4 - x^2) \implies y' = \dfrac{1}{4 - x^2}\left(4 - x^2\right)' = \dfrac{-2x}{4 - x^2}$

Example 5 of Logarithmic Function:

Given $f(x) = \ln(1 - x)$, find the equation of the tangent line at $x = 0$.

The derivative

$$f'(x) = \frac{1}{1 - x}\,(1 - x)' = \frac{-1}{1 - x} = \frac{1}{x - 1}$$

point at $x = 0$, $y = 0$

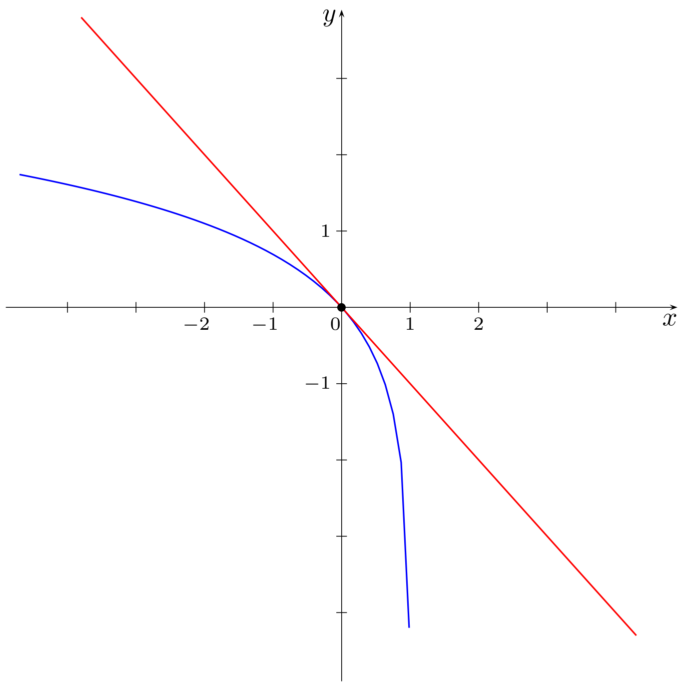
slope: $m = f(0) = -1$

equation of tangent line:

$y - 0 = -1(x - 0) \implies y = -x$

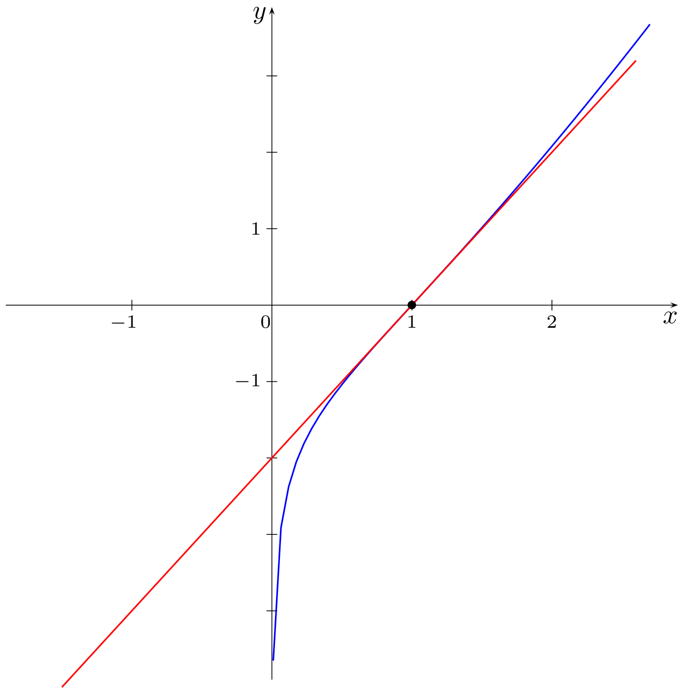
The graph shows the curve of $f(x)$ (blue curve) and the tangent line at $x = 0$ (red line)

Note: The graph is not part of the solution.
It is just visual !



Example 6 of Logarithmic Function:

Given $f(x) = (x+1)\ \ln(x)$, find the equation of the tangent line at $x = 1$.



The Product Rule: $(u.v)' = u'v + v'u$

$u = x+1 \rightarrow u' = 1\ ;\ v = \ln(x) \rightarrow v' = \dfrac{1}{x}$

$f'(x) = (1)\ \ln(x) + \dfrac{1}{x}\,(x+1) = \ln(x) + \dfrac{x+1}{x}$

point at $x = 1\,,\ y = 0$

slope: $m = f(1) = 2$

equation of tangent line:

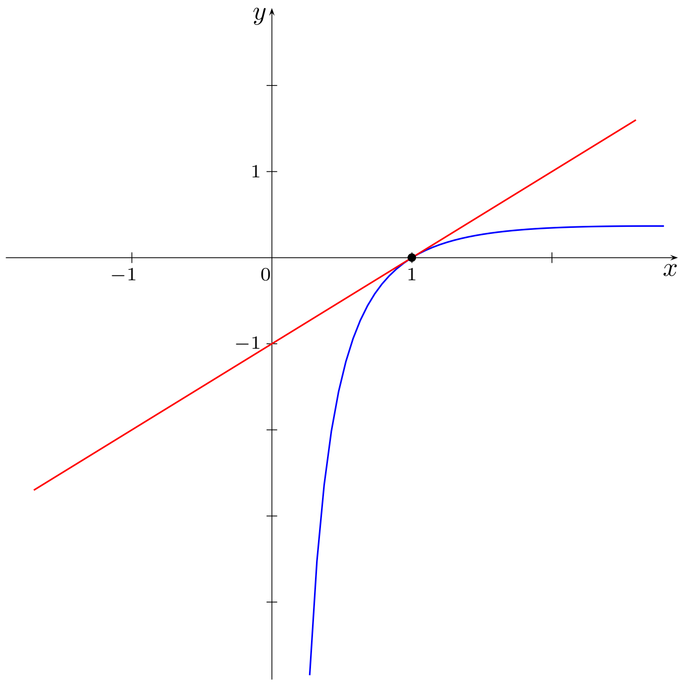
$y - 0 = 2(x-1) \implies y = 2x - 2$

The graph shows the curve of $f(x)$ (blue curve)
and the tangent line at $x = 1$ (red line)

Note: The graph is not part of the solution.
It is just visual !

Example 7 of Logarithmic Function:

Given $f(x) = \dfrac{\ln(x)}{x}$, find the equation of the tangent line at $x = 1$.



The Quotient Rule: $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - v'u}{v^2}$

$u = \ln(x) \rightarrow u' = \dfrac{1}{x}\ ;\ v = x \rightarrow v' = 1$

$f'(x) = \dfrac{\frac{1}{x}\,x - (1)\ \ln(x)}{x^2} = \dfrac{1 - \ln(x)}{x^2}$

point at $x = 1\,,\ y = 0$

slope: $m = f(1) = 1$

equation of tangent line:

$y - 0 = 1(x-1) \implies y = x - 1$

The graph shows the curve of $f(x)$ (blue curve)
and the tangent line at $x = 1$ (red line)

Note: The graph is not part of the solution.
It is just visual !

Example 8 of Logarithmic Function:

Given $f(x) = \ln(x^2+1)$, find the equation of the tangent line at $x = 1$.

The Chain Rule: $\Big(u(v)\Big)' = u'(v)\,.\,v'$

$u(v) = \ln(v) \rightarrow u'(v) = \dfrac{1}{v}\,.\,v'$

$v = x^2+1 \rightarrow v' = 2x$

$f'(x) = \dfrac{1}{x^2+1}\,.\,(2x) = \dfrac{2x}{x^2+1}$

point at $x = 1$, $y = \ln(2)$

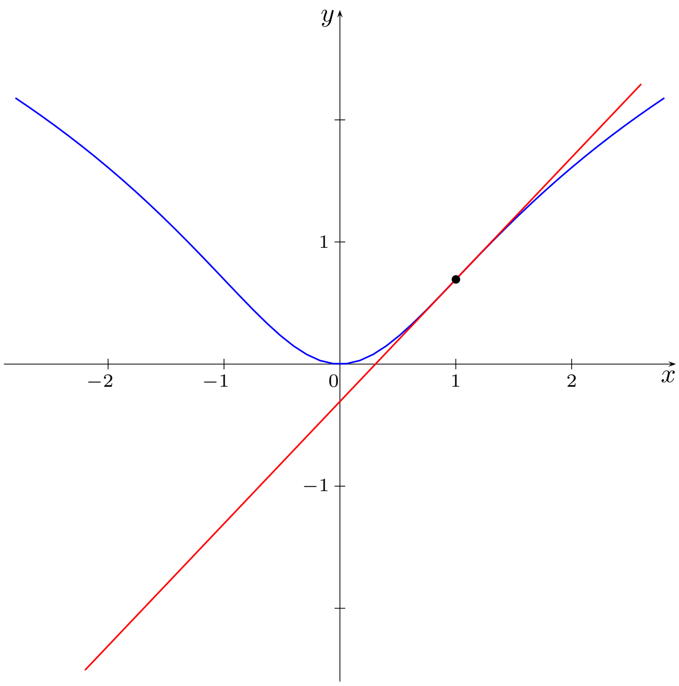
slope: $m = f(1) = 1$

equation of tangent line:

$y - \ln(2) = 1(x-0) \implies y = x - 1 + \ln(2)$

The graph shows the curve of $f(x)$ (blue curve) and the tangent line at $x = 1$ (red line)

<u>Note</u>: The graph is not part of the solution.
It is just visual !



Example 9 of Exponential & Logarithmic Functions:

Given $f(x) = e^{2\ \ln(x-1)}$, find the equation of the tangent line at $x = 2$.

Rewrite: $f(x) = e^{2\ \ln(x-1)} = (x-1)^2$

The Chain Rule: $\Big(u(v)\Big)' = u'(v)\,.\,v'$

$u(v) = (v)^2 \rightarrow u'(v) = 2v\,.\,v'$

$v = x-1 \rightarrow v' = 1$

$f'(x) = 2(x-1)\,.\,(1) = 2(x-1)$

point at $x = 2$, $y = 1$

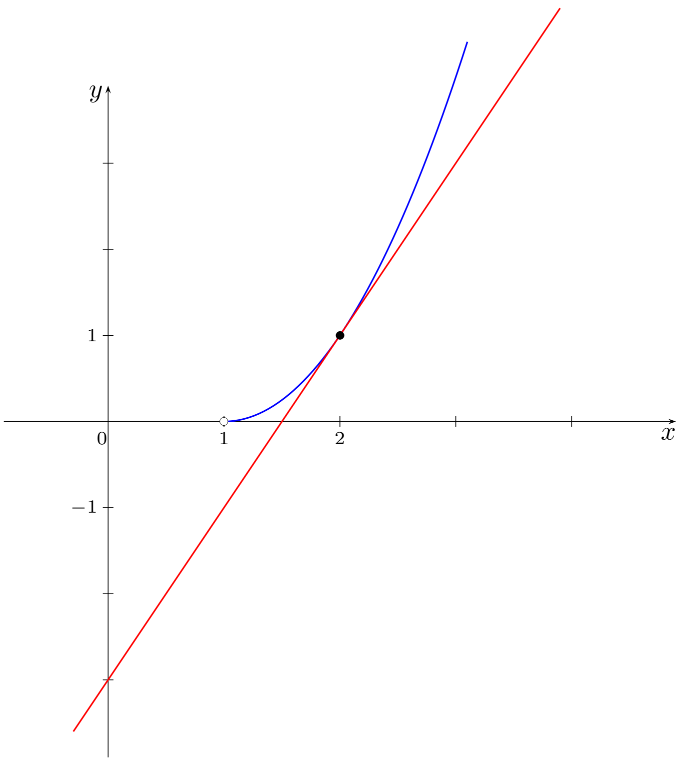
slope: $m = f(2) = 2$

equation of tangent line:

$y - 1 = 2(x-2) \implies y = 2x - 3$

The graph shows the curve of $f(x)$ (blue curve) and the tangent line at $x = 2$ (red line)

<u>Note</u>: The graph is not part of the solution.
It is just visual !



Example 10: Logarithmic Function

Find the derivative: $f(x) = \ln\left(\dfrac{1-2x}{x+3}\right)$

rewrite using log properties:

$f(x) = \ln(1-2x) - \ln(x+3)$

$$f'(x) = \frac{1}{1-2x}(-2) - \frac{1}{x+3}(1) = \frac{-2}{1-2x} - \frac{1}{x+3}$$

$$f'(x) = \frac{-2(x+3)-(1-2x)}{(1-2x)(x+3)} = \frac{-7}{(1-2x)(x+3)}$$

Example 11: Logarithmic Function

Find the derivative: $f(x) = \ln\left(\dfrac{\sqrt{x}}{x^3+1}\right)$

rewrite using log properties:

$f(x) = \ln(\sqrt{x}) - \ln\left(x^3+1\right) = \frac{1}{2}\ln(x) - \ln(x^3+1)$

$$f'(x) = \frac{1}{2}\frac{1}{x} - \frac{1}{x^3+1}(3x^2) = \frac{1}{2x} - \frac{3x^2}{x^3+1}$$

$$f'(x) = \frac{(x^3+1) - 3x^2(2x)}{2x(x^3+1)} = \frac{-5x^3+1}{2x(x^3+1)}$$

Example 12: Logarithmic Function

Find the derivative: $f(x) = \ln^2(4x+1)$

rewrite:

$$f(x) = \Big(\ln(4x+1)\Big)^2$$

$$f'(x) = 2\ln(4x+1)\ \frac{1}{4x+1}(4) = \frac{8}{4x+1}\ \ln(4x+1)$$