Math 201-103-RE

Practice Assignment 1

REVIEW - LIMITS - CONTINUITY

In this first practice assignment, you will review topics from sections 1.4, 1.5, 3.6, 8.3 (partial) and 1.6.
Try all problems for practice.
If you have difficulties, ask for help in class or in the instructor's office; all answers are shown in another file.

(1) Given that $g(x) = 6x - 2x^2$, find: (a) $g(-3)$; (b) $g(2)$; (c) $g(0)$; (d) $g\left(1+\sqrt{2}\right)$; (e) $g(-3a)$; (f) $g(2+t)$

(2) Given that $f(x) = 3 + 2x - x^2$, find: (a) $f(-1)$; (b) $f(3)$; (c) $f(0)$; (d) $f\left(-\sqrt{2}\right)$; (e) $f(2a)$; (f) $f(1-t)$

(3) Given that $f(x) = \dfrac{2-x}{3x}$, find: (a) $f(10)$; (b) $f(-6)$; (c) $f\left(-\frac{1}{2}\right)$; (d) $f(2\pi)$; (e) $f(5a)$; (f) $f(2+3t)$

(4) Given that $g(x) = \dfrac{3x}{1-x}$, find: (a) $g(4)$; (b) $g(-1)$; (c) $g\left(\frac{1}{3}\right)$; (d) $g\left(e^2\right)$; (e) $g(-2a)$; (f) $g(1-2t)$

(5) Given that $m(x) = \begin{cases} x^3 - 2x & \text{if } x \le 1 \\ 3x+1 & \text{if } x > 1 \end{cases}$, find:

(a) $m(-3)$; (b) $m(4)$; (c) $m(0)$; (d) $m(1)$; (e) $m(1.5)$; (f) $m(t^2+2)$ (graph for practice)

(6) Given that $f(x) = \begin{cases} 1 - x^2 & \text{if } x < -1 \\ \sqrt{x+2} & \text{if } x \ge -1 \end{cases}$, find:

(a) $f(2)$; (b) $f(-4)$; (c) $f(0)$; (d) $f(-1)$; (e) $f(-3.2)$; (f) $f(3t^2)$ (graph for practice)

(7) Given that $s(x) = \begin{cases} \frac{2}{\sqrt{x}} & \text{if } x > 2 \\ 2 - x & \text{if } x \le 2 \end{cases}$, find:

(a) $s(1)$; (b) $s(9)$; (c) $s(0)$; (d) $s(-5)$; (e) $s(-1.8)$; (f) $s(4t^2+3)$ (graph for practice)

(8) Given that $g(x) = \begin{cases} x^2 - 3x & \text{if } x \ge -2 \\ 1 - x^3 & \text{if } x < -2 \end{cases}$, find:

(a) $g(-4)$; (b) $g(1)$; (c) $g(0)$; (d) $g(4)$; (e) $g(2.1)$; (f) $g(t^2)$ (graph for practice)

(9) Given that $p(x) = \dfrac{(x-2)(3-x)}{2x}$, find: (a) $p(-2)$; (b) $p(3)$; (c) $p\left(\frac{1}{3}\right)$; (d) $p(1)$; (e) $p(-a)$; (f) $p\left(\frac{1}{a}\right)$

(10) Given that $h(x) = \dfrac{x+1}{(x-2)(x+2)}$, find: (a) $h(1)$; (b) $h(0)$; (c) $h\left(\frac{1}{4}\right)$; (d) $h(-3)$; (e) $h(2a)$; (f) $h(1-a)$

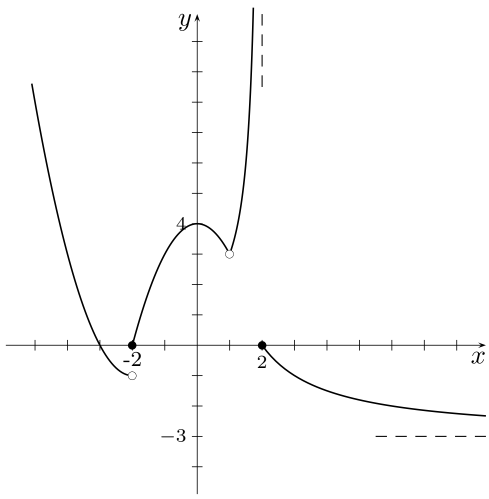
(11) Given the following graph of $f(x)$, find:

(a) $\lim\limits_{x\to-\infty} f(x)$ (b) $\lim\limits_{x\to-2^-} f(x)$

(c) $\lim\limits_{x\to1} f(x)$ (d) $\lim\limits_{x\to2^+} f(x)$

(e) $\lim\limits_{x\to2^-} f(x)$ (f) $\lim\limits_{x\to2} f(x)$

(g) $\lim\limits_{x\to+\infty} f(x)$



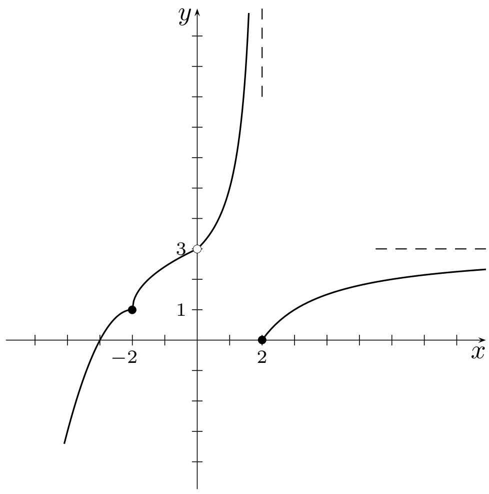
(12) Given the following graph of $f(x)$, find:

(a) $\lim\limits_{x\to-\infty} f(x)$ (b) $\lim\limits_{x\to-2} f(x)$

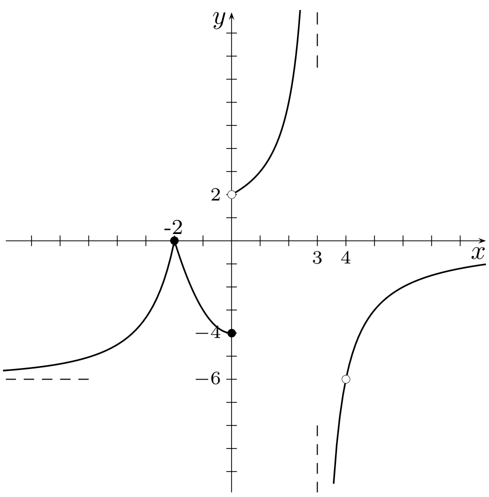
(c) $\lim\limits_{x\to0} f(x)$ (d) $\lim\limits_{x\to2^-} f(x)$

(e) $\lim\limits_{x\to2^+} f(x)$ (f) $\lim\limits_{x\to2} f(x)$

(g) $\lim\limits_{x\to+\infty} f(x)$



(13) Given the following graph of $f(x)$, find:



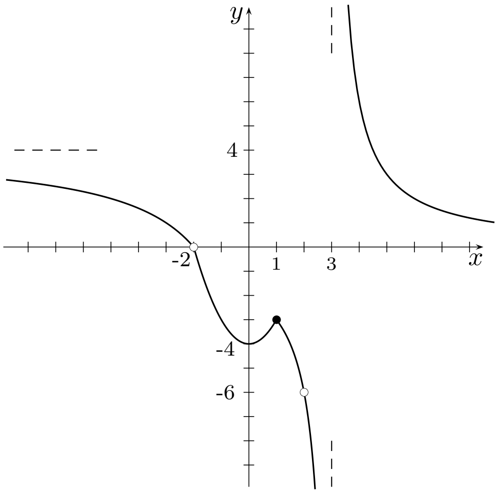
(a) $\lim\limits_{x\to-\infty} f(x)$ (b) $\lim\limits_{x\to-2} f(x)$

(c) $\lim\limits_{x\to0^-} f(x)$ (d) $\lim\limits_{x\to0^+} f(x)$

(e) $\lim\limits_{x\to3^+} f(x)$ (f) $\lim\limits_{x\to3^-} f(x)$

(g) $\lim\limits_{x\to+\infty} f(x)$

(14) Given the following graph of $f(x)$, find:



(a) $\lim_{x \to -\infty} f(x)$ (b) $\lim_{x \to 0} f(x)$

(c) $\lim_{x \to 1} f(x)$ (d) $\lim_{x \to 3^-} f(x)$

(e) $\lim_{x \to 3^+} f(x)$ (f) $\lim_{x \to 2} f(x)$

(g) $\lim_{x \to +\infty} f(x)$

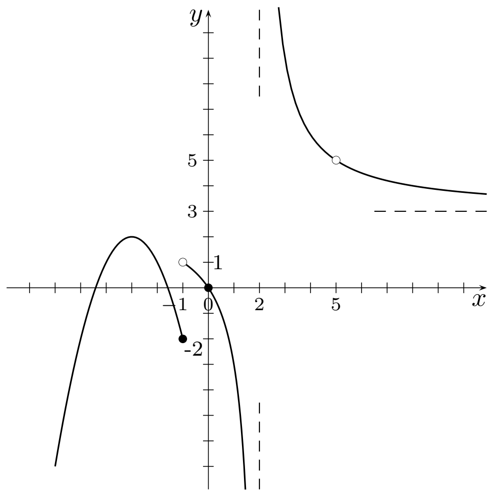
(15) Given the following graph of $f(x)$, find:

(a) $\lim_{x \to -\infty} f(x)$ (b) $\lim_{x \to -1^+} f(x)$

(c) $\lim_{x \to 5} f(x)$ (d) $\lim_{x \to 2^-} f(x)$

(e) $\lim_{x \to 2^+} f(x)$ (f) $\lim_{x \to -3} f(x)$

(g) $\lim_{x \to +\infty} f(x)$



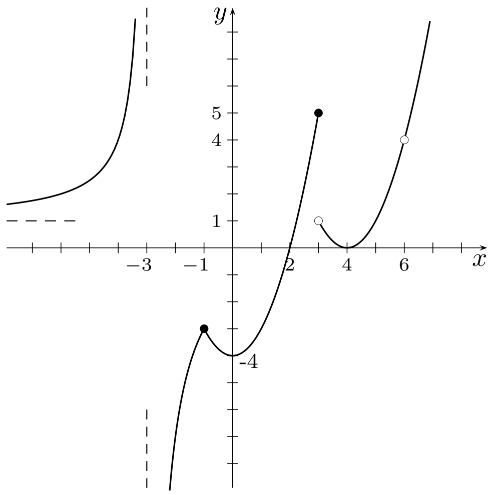
(16) Given the following graph of $f(x)$, find:

(a) $\lim_{x \to -\infty} f(x)$ (b) $\lim_{x \to -1} f(x)$

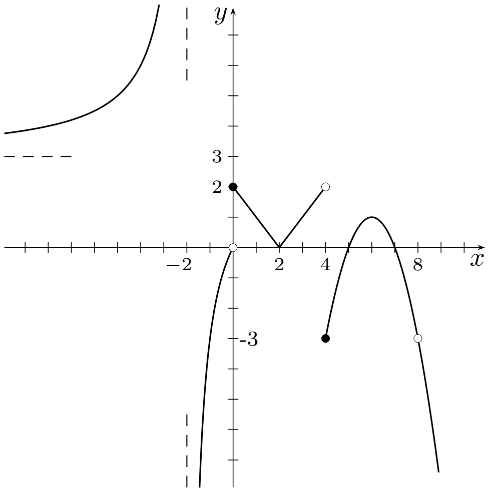
(c) $\lim_{x \to 1} f(x)$ (d) $\lim_{x \to 3^-} f(x)$

(e) $\lim_{x \to 3^+} f(x)$ (f) $\lim_{x \to 6} f(x)$

(g) $\lim_{x \to +\infty} f(x)$



(17) Given the following graph of $f(x)$, find:



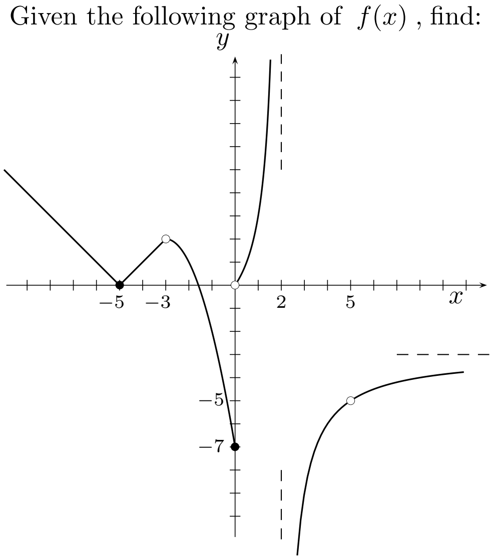
(a) $\lim_{x\to-\infty} f(x)$ (b) $\lim_{x\to-2^-} f(x)$

(c) $\lim_{x\to0} f(x)$ (d) $\lim_{x\to2} f(x)$

(e) $\lim_{x\to4^+} f(x)$ (f) $\lim_{x\to8} f(x)$

(g) $\lim_{x\to+\infty} f(x)$

(18) Given the following graph of $f(x)$, find:



(a) $\lim_{x\to-\infty} f(x)$ (b) $\lim_{x\to-3} f(x)$

(c) $\lim_{x\to0} f(x)$ (d) $\lim_{x\to2^-} f(x)$

(e) $\lim_{x\to2^+} f(x)$ (f) $\lim_{x\to2} f(x)$

(g) $\lim_{x\to+\infty} f(x)$

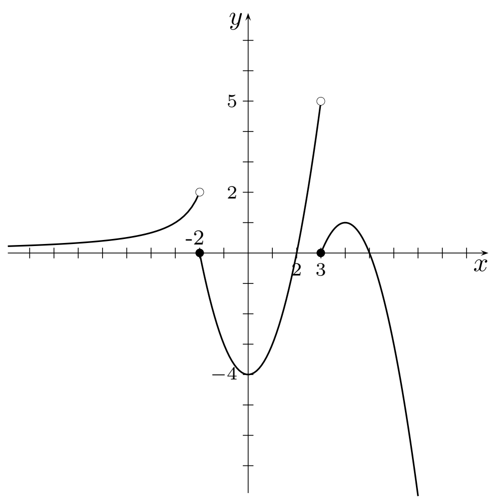
(19) Given the following graph of $f(x)$, find:

(a) $\lim_{x\to-\infty} f(x)$ (b) $\lim_{x\to-2^-} f(x)$

(c) $\lim_{x\to-2^+} f(x)$ (d) $\lim_{x\to3^-} f(x)$

(e) $\lim_{x\to2} f(x)$ (f) $\lim_{x\to0} f(x)$

(g) $\lim_{x\to+\infty} f(x)$



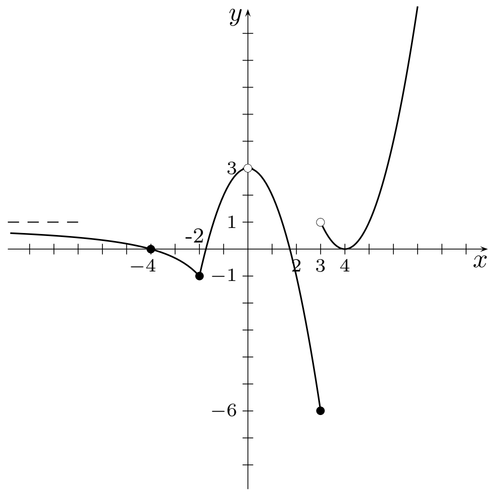
(20) Given the following graph of $f(x)$, find:

(a) $\lim_{x\to-\infty} f(x)$ (b) $\lim_{x\to-2} f(x)$

(c) $\lim_{x\to0} f(x)$ (d) $\lim_{x\to3^-} f(x)$

(e) $\lim_{x\to3^+} f(x)$ (f) $\lim_{x\to3} f(x)$

(g) $\lim_{x\to+\infty} f(x)$



(21) Evaluate $\lim_{x\to\frac{\pi}{2}} \dfrac{2+\sin x-\cos x}{4-\csc x}$ (22) Evaluate $\lim_{x\to0} \dfrac{\tan x+4\,\sec x}{2+x-\cos x}$ (23) Evaluate $\lim_{x\to\frac{\pi}{6}} \dfrac{2\,\cos 2x+1}{\sin x+2}$

(24) Evaluate $\lim_{x\to\pi^+} \dfrac{3-2\,\cot x}{\cos 2x}$ (25) Evaluate $\lim_{x\to0^+} \dfrac{3+\sec x}{\tan 2x}$

(26) Evaluate $\lim_{x\to\frac{\pi}{4}} \dfrac{3+\sin^2 x}{\tan x+2}$ (27) Evaluate $\lim_{x\to0} \dfrac{2+x+\cos x}{\sec x-2}$ (28) Evaluate $\lim_{x\to\frac{\pi}{3}} \dfrac{2-\sin\left(\frac{x}{2}\right)}{\cos x-1}$

(29) Evaluate $\lim_{x\to\frac{\pi}{2}^-} \dfrac{2+3\,\tan x}{4\,\sin x}$ (30) Evaluate $\lim_{x\to0^-} \dfrac{2(x-1)^2}{\sin 3x}$

(31) Given $f(x)=x-2x^2$ and $g(x)=2x+1$, evaluate $h(-2)$ where $h=f(g)$

(32) Given $f(x)=\sqrt{4-x^2}$ and $g(x)=3x^2$, evaluate $h(0)$ where $h=g(f)$

(33) Given $f(x)=\dfrac{4}{x^2+1}$ and $g(x)=\dfrac{3}{x}$, evaluate $h(3)$ where $h=f(g)$

(34) Given $f(x)=\dfrac{2}{x+4}$ and $g(x)=-x^2+1$, evaluate $h(-3)$ where $h=g(f)$

(35) Given $f(x)=\dfrac{4}{x^2}$ and $g(x)=2\sqrt{x}$, evaluate $h(2)$ where $h=f(g)$

(36) Given $f(x)=x^3-x-1$ and $g(x)=4-3x$, evaluate $h(-1)$ where $h=g(f)$

(37) Given $f(x)=\sqrt{x^3+1}$ and $g(x)=4-x^2$, evaluate $h(-2)$ where $h=f(g)$

(38) Given $f(x)=\dfrac{x}{x+2}$ and $g(x)=x+1$, evaluate $h(-4)$ where $h=g(f)$

(39) Given $f(x)=\dfrac{3}{1-x}$ and $g(x)=2+x^2$, evaluate $h(-2)$ where $h=f(g)$

(40) Given $f(x)=\dfrac{x+1}{x-2}$ and $g(x)=\dfrac{2}{x}$, evaluate $h(3)$ where $h=g(f)$

(41) Find value(s) of x at which the function is $r(x) = \dfrac{x^2+2x-3}{x^2-1}$ discontinuous.
(use continuity definition to prove it)

(42) Find value(s) of x at which the function is $q(x) = \begin{cases} 4x-1 & \text{for } x \le 1 \\ 2-x^2 & \text{for } x > 1 \end{cases}$ discontinuous.
(use continuity definition to prove it) (graph for practice)

(43) Find value(s) of x at which the function is $f(x) = \begin{cases} \sqrt{x}+2 & \text{for } x \ge 1 \\ 4-x+x^2 & \text{for } x < 1 \end{cases}$ discontinuous.
(use continuity definition to prove it) (graph for practice)

(44) Find value(s) of x at which the function is $g(x) = \dfrac{x+2}{x^2+x-2}$ discontinuous.
(use continuity definition to prove it)

(45) Find value(s) of x at which the function is $t(x) = \dfrac{3x^2}{6x+x^2}$ discontinuous.
(use continuity definition to prove it)

(46) Find value(s) of x at which the function is $g(x) = \dfrac{x^2-2x-3}{x^2-9}$ discontinuous.
(use continuity definition to prove it)

(47) Find value(s) of x at which the function is $h(x) = \begin{cases} x^2-1 & \text{for } x \le -2 \\ 3x+1 & \text{for } x > -2 \end{cases}$ discontinuous.
(use continuity definition to prove it) (graph for practice)

(48) Find value(s) of x at which the function is $G(x) = \begin{cases} x^2+x & \text{for } x \le -1 \\ x^3 & \text{for } x > -1 \end{cases}$ discontinuous.
(use continuity definition to prove it) (graph for practice)

(49) Find value(s) of x at which the function is $h(x) = \dfrac{x-3}{x^2-3x}$ discontinuous.
(use continuity definition to prove it)

(50) Find value(s) of x at which the function is $f(x) = \dfrac{x^2}{4x-x^2}$ discontinuous.
(use continuity definition to prove it)

(51) Evaluate $\displaystyle\lim_{z\to 3} \frac{z^2-z-6}{3z-z^2}$

(52) Evaluate $\displaystyle\lim_{x\to -4} \frac{1-\sqrt{x+5}}{x+4}$

(53) Evaluate $\displaystyle\lim_{x\to -\infty} \frac{4-x+6x^2}{2x^2-3x+4}$

(54) Evaluate $\displaystyle\lim_{x\to 1} \frac{\frac{1}{x+2}+\frac{1}{x-4}}{x-1}$

(55) Evaluate $\displaystyle\lim_{x\to 3} f(x)$ given $f(x) = \begin{cases} 2x-x^2 & \text{for } x < 3 \\ \sqrt{x+1} & \text{for } x \ge 3 \end{cases}$ (graph for practice)

(56) Evaluate $\displaystyle\lim_{z\to 1} \frac{z^2-3z+2}{z-z^2}$

(57) Evaluate $\displaystyle\lim_{x\to 3} \frac{4-\sqrt{x+13}}{x-3}$

(58) Evaluate $\displaystyle\lim_{x\to \infty} \frac{2+2x-4x^2}{2x^2+x+5}$

(59) Evaluate $\displaystyle\lim_{x\to -2} \frac{\frac{1}{x+3}+\frac{1}{x+1}}{x+2}$

(60) Evaluate $\lim_{x\to 2} f(x)$ given $f(x) = \begin{cases} x - 3x^2 & \text{for } x \le 2 \\ \sqrt{x-1} & \text{for } x > 2 \end{cases}$ (graph for practice)

(61) Use the definition of continuity to test $h(x) = \dfrac{4-x}{x^2 - 7x + 12}$ for discontinuity.

(62) Use the definition of continuity to test $v(x) = \dfrac{x^2 - x}{\sqrt{4 - x^2}}$ for discontinuity.

(63) Use the definition of continuity to test $f(x) = \begin{cases} 3x - 5 & \text{for } x < 2 \\ \sqrt{x-1} & \text{for } x > 2 \end{cases}$ for discontinuity. (graph for practice)

(64) Use the definition of continuity to test $f(x) = \begin{cases} 5x - 1 & \text{for } x > 1 \\ 2 & \text{for } x = 1 \\ 3x^2 + 1 & \text{for } x < 1 \end{cases}$ for discontinuity. (graph for practice)

(65) Use the definition of continuity to test $m(x) = \begin{cases} 11 - x^2 & \text{for } x \ge -3 \\ |x+1| & \text{for } x < -3 \end{cases}$ for discontinuity. (graph for practice)

(66) Use the definition of continuity to test $s(x) = \dfrac{2-x}{x^2 + 2x - 8}$ for discontinuity.

(67) Use the definition of continuity to test $u(x) = \dfrac{x+1}{\sqrt{x^2 - 9}}$ for discontinuity.

(68) Use the definition of continuity to test $N(x) = \begin{cases} x^2 + 3 & \text{for } x < -2 \\ -2x + 3 & \text{for } x > -2 \end{cases}$ for discontinuity. (graph for practice)

(69) Use the definition of continuity to test $p(x) = \begin{cases} \sqrt{x+6} & \text{for } x > 3 \\ 6 & \text{for } x = 3 \\ x^2 - 6 & \text{for } x < 3 \end{cases}$ for discontinuity. (graph for practice)

(70) Use the definition of continuity to test $q(x) = \begin{cases} |x-1| & \text{for } x \le -1 \\ 2x^2 & \text{for } x > -1 \end{cases}$ for discontinuity. (graph for practice)

(71) Evaluate $\lim_{x\to 1^+} \dfrac{3-x}{x-1}$

(72) Evaluate $\lim_{x\to 4} \dfrac{x-4}{\sqrt{x+21} - 5}$

(73) Evaluate $\lim_{x\to 3^-} \dfrac{6x - x^2}{x-3}$

(74) Evaluate $\lim_{x\to -1} \dfrac{3 - \frac{12}{x+5}}{x+1}$

(75) Evaluate $\lim_{x\to 2^+} \dfrac{4x}{2-x}$

(76) Evaluate $\lim_{x\to 2^-} \dfrac{x^2 - 1}{x-2}$

(77) Evaluate $\lim_{x\to -3} \dfrac{x+3}{3 - \sqrt{x+12}}$

(78) Evaluate $\lim_{x\to 4^+} \dfrac{9 - x^2}{x-4}$

(79) Evaluate $\lim_{x\to -2} \dfrac{2 - \frac{8}{x+6}}{x+2}$

(80) Evaluate $\lim_{x\to 3^-} \dfrac{2x}{2x-6}$

(81) Given that $f(x) = \begin{cases} x^2 + 4 & \text{if } x \le 1 \\ \sqrt{x^3 - 1} & \text{if } x > 1 \end{cases}$, find:

(a) $f(-4)$; (b) $f(-1)$; (c) $f(1)$; (d) $f(2)$; (e) $f(0)$; (f) $f(t^2+1)$ (graph for practice)

(82) Given $f(x) = \sqrt{4-x}$ and $g(x) = x^2 + 2$.

Evaluate (a) $g\big(f(x)\big)$; (b) $g\big(g(x)\big)$; (c) $f\big(g(x)\big)$; (d) $f\big(f(x)\big)$; (e) $f\big(f(-5)\big)$; (f) $g\big(g(-1)\big)$

(83) Use the definition of continuity to test $g(x) = \dfrac{x^2 - 3x - 4}{x^2 - 1}$ for discontinuity.

(84) Evaluate $\displaystyle\lim_{x \to -1^+} \frac{x^2 + 2x}{x^2 - 1}$

(85) Evaluate $\displaystyle\lim_{x \to 2^-} \frac{3x + 4}{x - 2}$

(86) Evaluate $\displaystyle\lim_{x \to \pi} \frac{3\sin 2x - 4\cos x}{2\tan\left(\frac{x}{4}\right)}$

(87) Evaluate $\displaystyle\lim_{x \to \infty} \frac{(2-x)(x+3)}{(3x-1)(x+1)}$

(88) Evaluate $\displaystyle\lim_{x \to 1} \frac{x - 1}{\frac{1}{4x} - \frac{1}{x+3}}$

(89) Evaluate $\displaystyle\lim_{x \to -2} \frac{x + 2}{\sqrt{x+3} - 1}$

(90) Evaluate $\displaystyle\lim_{x \to 0} \frac{4x - 5\cos 2x + 3}{3\tan x + 2\sec x}$

(91) Evaluate $\displaystyle\lim_{x \to \frac{\pi}{2}} \frac{2\cos 2x + 5\sin 3x}{4\csc x + 3\tan 2x}$

(92) Given $f(x) = \begin{cases} 2\sqrt{x+3} & \text{for } x < 1 \\ 2x - x^2 & \text{for } x \geq 1 \end{cases}$; evaluate: (a) $\displaystyle\lim_{x \to 1} f(x)$; (b) $\displaystyle\lim_{x \to -2} f(x)$; (c) $\displaystyle\lim_{x \to 2} f(x)$

(graph for practice)

(93) Given $g(x) = \begin{cases} 3x - 5 & \text{for } x < -1 \\ x^2 + 9 & \text{for } -1 \leq x < 1 \\ 2(x+4) & \text{for } x \geq 1 \end{cases}$; evaluate: (a) $\displaystyle\lim_{x \to -2} g(x)$; (b) $\displaystyle\lim_{x \to -1} g(x)$; (c) $\displaystyle\lim_{x \to 1} g(x)$; (d) $\displaystyle\lim_{x \to 0} g(x)$

(graph for practice)

(94) Use the definition of continuity to test $f(x) = \begin{cases} \frac{2}{x-1} & \text{for } x < 2 \\ \sqrt{6-x} & \text{for } x \geq 2 \end{cases}$ for discontinuity.

(95) Use the definition of continuity to test $g(x) = \begin{cases} \frac{3}{x^2-4} & \text{for } x > 1 \\ x^2 - 1 & \text{for } x \leq 1 \end{cases}$ for discontinuity.

(96) Given $f(x) = \frac{2}{x+1}$ and $g(x) = 4 - x$.

Evaluate (a) $f\big(g(x)\big)$; (b) $f\big(f(x)\big)$; (c) $g\big(f(2)\big)$; (d) $g\big(f(x)\big)$; (e) $g\big(g(-1)\big)$; (f) $f\big(f(0)\big)$

(97) Evaluate $\displaystyle\lim_{x \to 1^-} \frac{x^2 - 1}{x^2 + x - 2}$

(98) Evaluate $\displaystyle\lim_{x \to 3^-} \frac{\sqrt{3-x}}{x^2 - 9}$

(99) Evaluate $\displaystyle\lim_{x \to 0} \frac{\sin 2x - \tan 3x}{4x}$

(100) Evaluate $\displaystyle\lim_{x \to 0} \frac{2\cos x - \sin x - 2}{3\tan x}$