Math 201-103-RE

Practice Assignment 2

Derivatives & Applications

In this second practice assignment, you will review topics from sections 1.6, 2.1, 2.2, 2.3, 2.4 and 2.5.
Try all problems for practice.
If you have difficulties, ask for help in class or in the instructor's office; all answers are shown in another file.

(1) Use the definition of the derivative to find an equation of the tangent line to the graph of $f(x) = 6 + x - 3x^2$ at $x = 2$.

(2) Use the definition of the derivative to find an equation of the tangent line to the graph of $f(x) = 7x^2 - 6x + 3$ at $x = -1$.

(3) Use the definition of the derivative to find an equation of the tangent line to the graph of $f(x) = -2x^2 + 3x + 2$ at $x = 3$.

(4) Use the definition of the derivative to find an equation of the tangent line to the graph of $f(x) = 5x^2 - x + 4$ at $x = -2$.

(5) Use the definition of the derivative to find an equation of the tangent line to the graph of $f(x) = 4x^2 + 5x - 7$ at $x = -3$.

(6) Use the definition of the derivative to find an equation of the tangent line to the graph of $f(x) = 3x^2 + 8x - 6$ at $x = 1$.

(7) Given $f(x) = \begin{cases} k^2 - 3x & \text{if } x \le -1 \\ x^2 - 3kx & \text{if } x > -1 \end{cases}$, find the value(s) of k such that $f(x)$ is continuous for all x.

(8) Given $f(x) = \begin{cases} kx^2 + 4x & \text{if } x \ge -2 \\ k^2x + 2x^2 & \text{if } x < -2 \end{cases}$, find the value(s) of k such that $f(x)$ is continuous for all x.

(9) Given $f(x) = \begin{cases} 2k - 2x + x^2 & \text{if } x \le 1 \\ 2 + k^2x - 2x^2 & \text{if } x > 1 \end{cases}$, find the value(s) of k such that $f(x)$ is continuous for all x.

(10) Given $f(x) = \begin{cases} 2kx^2 + 3x & \text{if } x \ge 2 \\ 2k^2x + 3x & \text{if } x < 2 \end{cases}$, find the value(s) of k such that $f(x)$ is continuous for all x.

(11) Given $f(x) = \begin{cases} 10kx - 4x^2 & \text{if } x \le 3 \\ k^2x - x^2 & \text{if } x > 3 \end{cases}$, find the value(s) of k such that $f(x)$ is continuous for all x.

(12) Given $f(x) = \begin{cases} 5kx - x^2 & \text{if } x \ge -2 \\ k^2x^2 - (k+2)x & \text{if } x < -2 \end{cases}$, find the value(s) of k such that $f(x)$ is continuous for all x.

(13) Find the slope of the tangent line to the graph of $g(x) = 7x^2 - 2\sqrt{x} + \frac{16}{x} - 12$ at $x = 4$.

(14) Find the slope of the tangent line to the graph of $g(x) = 6x^3 - 4\sqrt[3]{x} - \frac{3}{x^2} + 3$ at $x = -1$.

(15) Find the slope of the tangent line to the graph of $g(x) = \dfrac{4}{\sqrt[3]{x}} - 12x^2 - 6\sqrt{x} + 7$ at $x = 1$.

(16) Find the slope of the tangent line to the graph of $g(x) = 18x^4 + 6\sqrt[4]{x^3} - \frac{5}{x^2} + 10$ at $x = 1$.

(17) Find the slope of the tangent line to the graph of $g(x) = 6x - \frac{15}{x^4} + 4x^2 + 13$ at $x = 2$.

(18) Find the slope of the tangent line to the graph of $g(x) = 7x + \frac{12}{x^3} - 16x^2 + 25$ at $x = -1$.

(19) Let the revenue R in dollars obtained in selling x units of an item be $R(x) = 3x(2 + x^2) + 600$. Find the marginal revenue when production is 35 units. Interpret the result.

(20) Let the average cost $\overline{C}$ in dollars per unit obtained in selling x units of an item be $\overline{C}(x) = -2x^2 + 4 + \frac{100}{x}$. Find the marginal cost when production is 50 units. Interpret the result.

(21) Let the demand in dollars/unit be $p(x) = 3x + 5x^2$ and the average cost in dollars/unit be $\overline{C}(x) = 5 + 4x$ obtained in selling x units. Find the marginal profit when production is 20 units. Interpret the result.

(22) Let the demand in dollars/unit obtained in selling x units of an item be $p(x) = 4x + 10x^2$. Find the marginal revenue when production is 35 units. Interpret the result.

(23) Let the average cost in dollars/unit obtained in selling x units of an item be $\overline{C}(x) = 5(4 - 3x) + \frac{75}{x}$. Find the marginal cost when production is 12 units. Interpret the result.

(24) Let the price function in dollars per unit obtained in selling x units of an item be $p = 8x - 4 + \frac{80}{x}$. Find the marginal revenue when production is 15 units. Interpret the result.

(25) Find the slope of the tangent line to the graph of $f(x) = \left(x^2 + 1\right)\left(1 - x^3\right)$ at $x = 2$.

(26) Find the slope of the tangent line to the graph of $f(x) = \left(3x^2 + x + 2\right)(1 + 3x)$ at $x = -1$.

(27) Find the slope of the tangent line to the graph of $f(x) = \left(x^2 - 2x\right)\left(5 - x + 2x^2\right)$ at $x = 3$.

(28) Find the slope of the tangent line to the graph of $f(x) = \left(5 + x - x^2\right)\left(2x^2 - 3x + 1\right)$ at $x = 1$.

(29) Find the slope of the tangent line to the graph of $f(x) = (2x + \sqrt{x})\left(10\sqrt{x} - 3x^2\right)$ at $x = 1$.

(30) Find the slope of the tangent line to the graph of $f(x) = (3x - 2\sqrt{x})\left(x^3 - x\right)$ at $x = 4$.

(31) Find the equation of the tangent line to the graph of $f(x) = \dfrac{2x}{x + 3}$ at $x = -2$.

(32) Find the equation of the tangent line to the graph of $f(x) = \dfrac{1 - 3x}{x - 1}$ at $x = 2$.

(33) Find the equation of the tangent line to the graph of $f(x) = \dfrac{2+\sqrt{x}}{2-\sqrt{x}}$ at $x = 1$.

(34) Find the equation of the tangent line to the graph of $f(x) = \dfrac{4\sqrt{x}}{x-3}$ at $x = 9$.

(35) Find the equation of the tangent line to the graph of $f(x) = \dfrac{2x+3}{4+\sqrt{x}}$ at $x = 1$.

(36) Find the slope of the tangent line to the graph of $g(x) = \dfrac{6}{(2+x-x^2)^3}$ at $x = 1$.

(37) Find the slope of the tangent line to the graph of $g(x) = \sqrt[3]{(4-4x-x^2)^4}$ at $x = 2$.

(38) Find the slope of the tangent line to the graph of $g(x) = \dfrac{9}{\sqrt{2x^2-4x+3}}$ at $x = -1$.

(39) Find the slope of the tangent line to the graph of $g(x) = \left(x^4-2x^2-9\right)^4$ at $x = 2$.

(40) Find the slope of the tangent line to the graph of $g(x) = \sqrt[5]{x^3-3x+3}$ at $x = -2$.

(41) Use the definition of the derivative to find an equation of the tangent line to the graph of $f(x) = 4-2x-x^2$ at $x = 1$.

(42) Use the definition of the derivative to find an equation of the tangent line to the graph of $f(x) = 2x^2+4x+1$ at $x = -2$.

(43) Use the definition of the derivative to find an equation of the tangent line to the graph of $f(x) = -4x^2+5x+6$ at $x = -1$.

(44) Use the definition of the derivative to find an equation of the tangent line to the graph of $f(x) = 3x^2-7x+2$ at $x = 2$.

(45) Use the definition of the derivative to find an equation of the tangent line to the graph of $f(x) = 6x^2-9x+5$ at $x = 3$.

(46) Use the definition of the derivative to find an equation of the tangent line to the graph of $f(x) = 5x^2-3x+6$ at $x = -3$.

(47) Given $f(x) = \begin{cases} k^2x - x^2 & \text{if } x \geq 2 \\ kx & \text{if } x < 2 \end{cases}$, find the value(s) of k such that $f(x)$ is continuous for all x.

(48) Given $f(x) = \begin{cases} 6kx + x^2 & \text{if } x \leq 1 \\ (kx)^2 + 6x & \text{if } x > 1 \end{cases}$, find the value(s) of k such that $f(x)$ is continuous for all x.

(49) Given $f(x) = \begin{cases} 6k - 3x + kx^2 & \text{if } x \geq -1 \\ 3x^2 - k^2x - 8 & \text{if } x < -1 \end{cases}$, find the value(s) of k such that $f(x)$ is continuous for all x.

(50) Given $f(x) = \begin{cases} 8kx - 3x^2 & \text{if } x \le -2 \\ 2k^2 + 5x^2 & \text{if } x > -2 \end{cases}$, find the value(s) of k such that $f(x)$ is continuous for all x.

(51) Given $f(x) = \begin{cases} 4kx^2 + 2x^4 & \text{if } x \ge -3 \\ 2k^2x^2 - 6x & \text{if } x < -3 \end{cases}$, find the value(s) of k such that $f(x)$ is continuous for all x.

(52) Given $f(x) = \begin{cases} 3kx^2 - 4x & \text{if } x \le -1 \\ 2k^2x + 4kx - 2x^2 & \text{if } x > -1 \end{cases}$, find the value(s) of k such that $f(x)$ is continuous for all x.

(53) Find the slope of the tangent line to the graph of $g(x) = 5x^3 - 4\sqrt{x} + \frac{2}{\sqrt{x}} - 8$ at $x = 1$.

(54) Find the slope of the tangent line to the graph of $g(x) = 8x^4 - 6\sqrt[3]{x^2} - \frac{4}{x^3} + 9$ at $x = -1$.

(55) Find the slope of the tangent line to the graph of $g(x) = \dfrac{2}{\sqrt[3]{x^5}} + 8x^{1/2} - 2x^4 + 5$ at $x = 1$.

(56) Find the slope of the tangent line to the graph of $g(x) = \dfrac{4}{\sqrt{x^3}} - 12\sqrt[3]{x^4} + 32x^2 - 25$ at $x = 1$.

(57) Find the slope of the tangent line to the graph of $g(x) = 3x^2 - \frac{5}{x^2} + 5x - 20$ at $x = 2$.

(58) Find the slope of the tangent line to the graph of $g(x) = 12x - \frac{8}{x^4} + 10x^3 - 18$ at $x = -2$.

(59) Let the revenue R in dollars obtained in selling x units of an item be $R(x) = 4x(3 + x^2) + 800$. Find the marginal revenue when production is 40 units. Interpret the result.

(60) Let the average cost $\overline{C}$ in dollars per unit obtained in selling x units of an item be $\overline{C}(x) = -3x + 6 + \frac{220}{x}$. Find the marginal cost when production is 15 units. Interpret the result.

(61) Let the demand in dollars/unit be $p(x) = 2x + 7x^2$ and the average cost in dollars/unit be $\overline{C}(x) = 4 + 3x$ obtained in selling x units. Find the marginal profit when production is 25 units. Interpret the result.

(62) Let the demand in dollars/unit obtained in selling x units of an item be $p(x) = 6x + 4x^2$. Find the marginal revenue when production is 20 units. Interpret the result.

(63) Let the average cost in dollars/unit obtained in selling x units of an item be $\overline{C}(x) = 6(3 - 7x) + \frac{150}{x}$. Find the marginal cost when production is 22 units. Interpret the result.

(64) Let the price function in dollars per unit obtained in selling x units of an item be $p = 12x - 8 + \frac{90}{x}$. Find the marginal revenue when production is 9 units. Interpret the result.

(65) Find the slope of the tangent line to the graph of $f(x) = \left(x^2 + x\right)\left(2 - x^2\right)$ at $x = -2$.

(66) Find the slope of the tangent line to the graph of $f(x) = \left(4x^2 + 3x + 1\right)(2 + 5x)$ at $x = 1$.

(67) Find the slope of the tangent line to the graph of $f(x) = \left(x - x^3\right)\left(1 + x - x^2\right)$ at $x = -1$.

(68) Find the slope of the tangent line to the graph of $f(x) = \left(2 + 3x + x^2\right)\left(4x^2 - 2x + 3\right)$ at $x = -1$.

(69) Find the slope of the tangent line to the graph of $f(x) = (4x + 2\sqrt{x})\left(6\sqrt{x} - 4x^2\right)$ at $x = 4$.

(70) Find the slope of the tangent line to the graph of $f(x) = (4x - 6\sqrt{x})\left(5x - x^3\right)$ at $x = 1$.

(71) Find the equation of the tangent line to the graph of $f(x) = \dfrac{3x}{x - 4}$ at $x = 5$.

(72) Find the equation of the tangent line to the graph of $f(x) = \dfrac{2 - 5x}{x - 3}$ at $x = 2$.

(73) Find the equation of the tangent line to the graph of $f(x) = \dfrac{\sqrt{x} + 1}{3 - \sqrt{x}}$ at $x = 4$.

(74) Find the equation of the tangent line to the graph of $f(x) = \dfrac{3\sqrt{x}}{2 - x}$ at $x = 1$.

(75) Find the equation of the tangent line to the graph of $f(x) = \dfrac{5x - 1}{2\sqrt{x} - 3}$ at $x = 4$.

(76) Find the slope of the tangent line to the graph of $g(x) = \dfrac{4}{(3 + 2x + x^2)^2}$ at $x = 1$.

(77) Find the slope of the tangent line to the graph of $g(x) = \sqrt[5]{(x^3 - 4x + 1)^3}$ at $x = 2$.

(78) Find the slope of the tangent line to the graph of $g(x) = \dfrac{12}{\sqrt{9 - 2x - x^2}}$ at $x = -2$.

(79) Find the slope of the tangent line to the graph of $g(x) = \left(8 + 5x + 4x^3\right)^5$ at $x = -1$.

(80) Find the slope of the tangent line to the graph of $g(x) = \sqrt[3]{2x^2 - 6x + 1}$ at $x = 3$.

(81) Given $f(x) = \begin{cases} cx^2 + 2 & \text{if } x > 2 \\ -5x + c^2 & \text{if } x \le 2 \end{cases}$, find the value(s) of c such that $f(x)$ is continuous for all x.

(82) Given $f(x) = \begin{cases} (x + k)^2 & \text{if } x > -1 \\ 2kx + 5 & \text{if } x \le -1 \end{cases}$, find the value(s) of k such that $f(x)$ is continuous for all x.

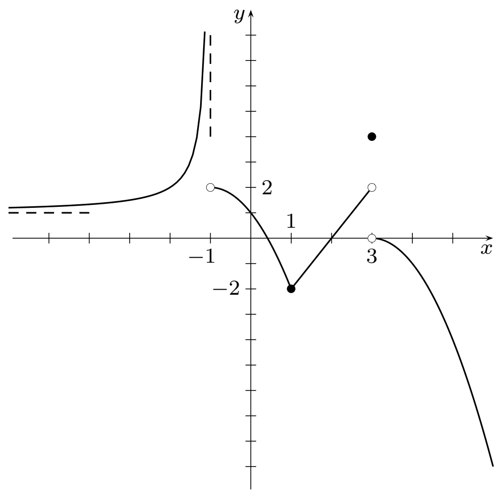
(83) Let the revenue R in dollars obtained in selling x units of an item be $R(x) = 4x^2 + 5x + 100$.
Find the marginal demand when production is 50 units. Interpret the result.

(84) Let the cost C in dollars obtained in producing x units of an item be $C(x) = 0.1x^3 + 50x + 200$.
Find the marginal average cost when production is 20 units. Interpret the result.

(85) Find the equation of the tangent line to the graph of $f(x) = \left(4x - x^2\right)\left(x^3 + 4\right)$ at $x = 1$.

(86) Find the equation of the tangent line to the graph of $f(x) = \frac{24}{\sqrt{x}} + \frac{16}{x^2} + 3x$ at $x = 4$.

(87) Given the graph of $f(x)$,



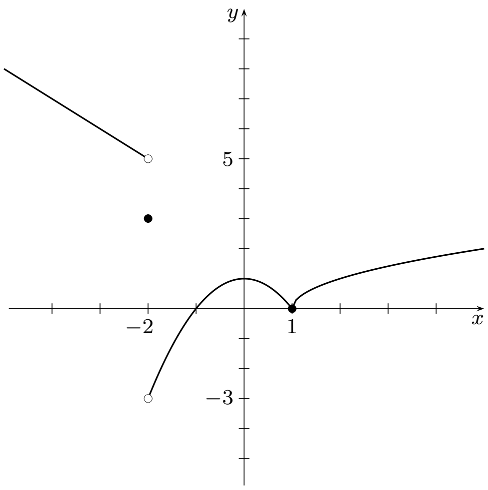
(a) Give the interval(s) where the slope of the tangent line to the curve of $f(x)$ is negative.

(b) Locate the x-value(s) where $f(x)$ is continuous but not differentiable.

(c) Give the interval(s) where $f(x)$ is continuous.

(d) Find the values of $f(-1)$; $f(0)$; $f(1)$; $f(3)$

(88) Given the graph of $f(x)$,



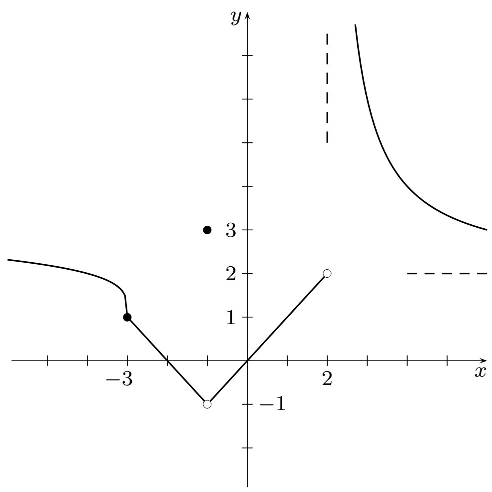
(a) Give the interval(s) where the slope of the tangent line to the curve of $f(x)$ is positive.

(b) Locate the x-value(s) where $f(x)$ is continuous but not differentiable.

(c) Give the interval(s) where $f(x)$ is continuous.

(d) Find the values of $f(-2)$; $f(0)$; $f(1)$; $f(-1)$

(89) Given the graph of $f(x)$,



(a) Give the interval(s) where the slope of the tangent line to the curve of $f(x)$ is negative.

(b) Locate the x-value(s) where $f(x)$ is continuous but not differentiable.

(c) Give the interval(s) where $f(x)$ is continuous.

(d) Find the values of $f(-3)$; $f(-2)$; $f(-1)$; $f(2)$

(90) Find the equation of the tangent line to the graph of $f(x) = \dfrac{x^2}{x-2}$ at $x = 3$.

(91) Find the equation of the tangent line to the graph of $f(x) = \left(x^3 - 2x^2 + 3x - 1\right)^{3/2}$ at $x = 1$.

(92) Use the definition of the derivative to find the slope of the tangent line to the graph of $f(x) = \frac{4}{x+1}$ at $x = 3$.

(93) Use the definition of the derivative to find the slope of the tangent line to the graph of $f(x) = 6\sqrt{x+2}$ at $x = 2$.

(94) If $f(1) = 5$, $f'(1) = -2$ and $g(x) = x^3 \,.\, f(x)$, then find $g'(1)$

(95) If $h(2) = 4$, $h'(2) = -3$ and $f(x) = \dfrac{2\,h(x)}{x^2}$, then find $f'(2)$

(96) Given $f(x) = x^3 + 6x^2 - 15x + 4$, find the x-value(s) such that the tangent line to the curve of $f(x)$ is horizontal.

(97) Find the point(s) on the curve of $f(x) = 2x^3 + 15x^2 - 140x + 10$ such that the slope of the tangent line is 4.

(98) If $g(-1) = -4$, $g'(1) = 7$ and $f(x) = g\left(x^2\right)$, then find $f'(-1)$

(99) Given $f(x) = \dfrac{x^2}{x+4}$, find the point(s) such that the tangent line to the curve of $f(x)$ is horizontal.

(100) Given $f(x) = \dfrac{\sqrt{x}}{x^2+3}$, find the point(s) such that the tangent line to the curve of $f(x)$ is horizontal.