Math 201-103-RE

Practice Assignment 5

Applications of the Derivative

In this fifth practice assignment, you will review topics from sections 3.1, 3.2, 3.3, 3.4, 3.5, 3.6 and 3.7
Try all problems for practice.
If you have difficulties, ask for help in class or in the instructor's office; all answers are shown in another file.

(1) Find the critical numbers and the open intervals on which the function $f(x) = -\frac{x^3}{3} + x^2 + 3x + 4$ is increasing or decreasing.

(2) Find the critical numbers and the open intervals on which the function $f(x) = \dfrac{5x^2 + 5}{x}$ is increasing or decreasing.

(3) Find the critical numbers and the open intervals on which the function $f(x) = \dfrac{-3x^2 - 12}{x}$ is increasing or decreasing.

(4) Find the critical numbers and the open intervals on which the function $f(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2 + 4$ is increasing or decreasing.

(5) Find the critical numbers and the open intervals on which the function $f(x) = \dfrac{3x^2 - 5x + 27}{x}$ is increasing or decreasing.

(6) Find all relative extrema of the function $f(x) = \dfrac{x^2 - 2x + 9}{2 - x}$

(7) Find all relative extrema of the function $f(x) = \frac{1}{2}x^4 + 2x^3 + 2$

(8) Find all relative extrema of the function $f(x) = \dfrac{4x^2 + 9x + 9}{x + 1}$

(9) Find all relative extrema of the function $f(x) = \dfrac{2x^3 - 4}{x}$

(10) Find the absolute extrema of the function $f(x) = \frac{1}{2}x^4 - 4x^2 + 5$ on the interval $[1, 3]$

(11) Find the absolute extrema of the function $f(x) = \dfrac{-x^3 - 4}{x^2}$ on the interval $[1, 4]$

(12) Find the absolute extrema of the function $f(x) = \frac{5}{2}x^4 - \frac{20}{3}x^3 + 6$ on the interval $[-1, 3]$

(13) Find the absolute extrema of the function $f(x) = \frac{3}{2}x^4 - 4x^3 + 4$ on the interval $[0, 3]$

(14) Find the intervals on which the graph of $f(x) = \dfrac{3x^3 + 10x - 24}{2x}$ is concave upward and those on which it is concave downward. Find the points of inflection.

(15) Find the intervals on which the graph of $f(x) = -x^4 + 2x^3 + 5x$ is concave upward and those on which it is concave downward. Find the points of inflection.

(16) Find the intervals on which the graph of $f(x) = \dfrac{x^3 - x^2 - 8}{x - 1}$ is concave upward and those on which it is concave downward. Find the points of inflection.

(17) Find all relative extrema using the second derivative test of $f(x) = \frac{1}{4}x^4 - \frac{5}{3}x^3 + 2x^2 + \frac{35}{3}$

(18) Find all relative extrema using the second derivative test of $f(x) = \dfrac{6x^3 + 96}{x}$

(19) Find all relative extrema using the second derivative test of $f(x) = \dfrac{10\left(x^2 + x + 4\right)}{x + 1}$

(20) Use the derivative method to sketch the graph of
$f(x) = \dfrac{(x-2)(2x-1)}{(x+1)^2}$ with $f'(x) = \dfrac{9(x-1)}{(x+1)^3}$ and $f''(x) = \dfrac{18\,(2-x)}{(x+1)^4}$

(21) Use the derivative method to sketch the graph of
$f(x) = \left(\dfrac{x+2}{x-2}\right)^2$ with $f'(x) = \dfrac{-8(x+2)}{(x-2)^3}$ and $f''(x) = \dfrac{16\,(x+4)}{(x-2)^4}$

(22) Use the derivative method to sketch the graph of
$f(x) = \dfrac{6x^2}{4-x^2}$ with $f'(x) = \dfrac{48x}{(4-x^2)^2}$ and $f''(x) = \dfrac{48\,(3x^2+4)}{(4-x^2)^3}$

(23) Use the derivative method to sketch the graph of
$f(x) = \dfrac{6}{x^2+4x}$ with $f'(x) = \dfrac{-12(x+2)}{(x^2+4x)^2}$ and $f''(x) = \dfrac{12\,(3x^2+12x+16)}{(x^2+4x)^3}$

(24) Use the derivative method to sketch the graph of
$f(x) = \frac{1}{5}x^5 - \frac{2}{3}x^3 + x$ with $f'(x) = (x+1)^2\,(x-1)^2$ and $f''(x) = 4x\;(x+1)\,(x-1)$

(25) Use the derivative method to sketch the graph of
$f(x) = x^3 + 9x^2 + 120$ with $f'(x) = 3x(x+6)$ and $f''(x) = 6x + 18$

(26) Use the derivative method to sketch the graph of
$f(x) = (x-1)^4\,(3x+2)$ with $f'(x) = 5\;(x-1)^3\,(3x+1)$ and $f''(x) = 60x\,(x-1)^2$

(27) Use the derivative method to sketch the graph of
$f(x) = x + \dfrac{1}{x+2}$ with $f'(x) = \dfrac{(x+1)(x+3)}{(x+2)^2}$ and $f''(x) = \dfrac{2}{(x+2)^3}$

(28) Use the derivative method to sketch the graph of
$f(x) = \dfrac{x^2}{x-1}$ with $f'(x) = \dfrac{x(x-2)}{(x-1)^2}$ and $f''(x) = \dfrac{2}{(x-1)^3}$

(29) A pear plantation contains 90 trees per acre. The average yield is 300 pears per tree. The farmer wishes to plant more trees to maximize the yield per acre. From past evidence the farmer has determined that for each additional tree planted per acre, the yield per tree is reduced by 3 pears.
Find the number of trees per acre that will maximize the yield per acre.

(30) A Tire Company has 20 stores on the island of Montreal, each of which has an average income of \$7000 per week. After study, the company notices that for each new store opened, the average weekly income drops \$70. How many new stores should be opened to maximize income?

(31) The average cost in dollars/unit to manufacture a product is $\overline{C} = -\frac{1}{3}x^2 - 30x - 1600 + \frac{50\,000}{x}$.
How many units should be made in order to minimize the cost and what is the minimum cost?

(32) The selling price in dollars/unit to sell a product is $p = -\frac{1}{3}x^2 + \frac{17}{2}x + 270$.
How many units should be made in order to maximize the revenue and what is the maximum revenue?

(33) A company manufactures and sells x units of a product per week. If the weekly average cost in dollars per unit is $\overline{C} = x^2 + 8x + 20$ and the selling price in dollars per unit is $p = \frac{2}{3}x^2 + 20x + 200$. Find for each week, the maximum revenue, the maximum profit, the production level that will realize the maximum profit and the price that the company should charge for each unit.

(34) If the average manufacturing cost (in dollars per unit) of a product is given by $\overline{C} = \frac{5}{4}x^3 + 5x^2 + 3x$ where x is the number of units manufactured and the selling price in dollars per unit is $p = x^3 + 10x^2 + 30x$, what is the production to maximize the profit? What is the maximum profit?

(35) The demand function for a product is given by $p = \dfrac{5x}{x-12}$ where p is the price per unit when x units are demanded.
(a) Determine the intervals of elasticity of demand
(b) If the price of the product at \$25 decreases by 4%, what is the approximate percentage in demand?
(c) Will the total revenue increase, decrease or remain constant at $p = \$25$?

(36) The demand function for a product is given by $p = \sqrt{400 - x}$ where p is the price per unit when x units are demanded.
(a) Determine the intervals of elasticity of demand
(b) If the price of the product at \$10 decreases by 3%, what is the approximate percentage in demand?
(c) Will the total revenue increase, decrease or remain constant at $p = \$10$?

(37) The demand function for a product is given by $p = -x^2 + 45x + 1200$ where p is the price per unit when x units are demanded.
(a) Determine the intervals of elasticity of demand
(b) If the price of the product at \$1650 decreases by 3%, what is the approximate percentage in demand?
(c) Will the total revenue increase, decrease or remain constant at $p = \$1650$?

(38) A rancher has 45000 square feet of land which he will use to subdivide into 2 equal rectangular fields with the inner fence costs \$20 per linear foot and the outer fence costs \$40 per linear foot.
Find the minimum cost of the fence.

(39) A company wants to enclose with a fence a storage area next to a wall of the building with an area of 6400 square meters. The fence opposite the wall of the building costs \$10 per linear meter and the fence of the other two sides costs \$20 per linear meter.
Find the dimensions of the storage area to minimize the cost of the fence.

(40) The demand function for a product is given by $p = \dfrac{x}{\sqrt{x+20}}$ where p is the price per unit when x units are demanded.
(a) Determine the intervals of elasticity of demand
(b) If the price of the product at \$1 increases by 9%, what is the approximate percentage in demand?
(c) Will the total revenue increase, decrease or remain constant at $p = \$1$?

(41) Find the critical numbers and the open intervals on which the function $f(x) = -\frac{1}{3}x^3 - \frac{1}{2}x^2 + 6x + 3$ is increasing or decreasing.

(42) Find the critical numbers and the open intervals on which the function $f(x) = -\dfrac{6x^2+24}{x}$ is increasing or decreasing.

(43) Find the critical numbers and the open intervals on which the function $f(x) = \dfrac{-5x^3+2x+8}{x^2}$ is increasing or decreasing.

(44) Find the critical numbers and the open intervals on which the function $f(x) = \frac{1}{4}x^4 - \frac{5}{3}x^3 + 2x^2 + 3$ is increasing or decreasing.

(45) Find the critical numbers and the open intervals on which the function $f(x) = \dfrac{-2x^2+3x-8}{x}$ is increasing or decreasing.

(46) Find all relative extrema of the function $f(x) = \dfrac{x^2-x+4}{x-1}$

(47) Find all relative extrema of the function $f(x) = \frac{3}{4}x^4 - 3x^3 + 4$

(48) Find all relative extrema of the function $f(x) = \dfrac{2x^2+7x+8}{x+2}$

(49) Find all relative extrema of the function $f(x) = \dfrac{3x^3+6}{x}$

(50) Find the absolute extrema of the function $f(x) = 2x^4 - 36x^2 + 20$ on the interval $[-4,-1]$

(51) Find the absolute extrema of the function $f(x) = \dfrac{2x^3+27}{2x^2}$ on the interval $[2,5]$

(52) Find the absolute extrema of the function $f(x) = \frac{40}{3}x^3 - 2x^4 + 10$ on the interval $[-1,6]$

(53) Find the absolute extrema of the function $f(x) = -\frac{4}{5}x^5 + \frac{1}{2}x^4 + 8$ on the interval $[-2,1]$

(54) Find the intervals on which the graph of $f(x) = \dfrac{x^3 + 4x + 27}{x}$ is concave upward and those on which it is concave downward. Find the points of inflection.

(55) Find the intervals on which the graph of $f(x) = x^4 + 4x^3 - 5x$ is concave upward and those on which it is concave downward. Find the points of inflection.

(56) Find the intervals on which the graph of $f(x) = \frac{3}{2}\,x^2 + \dfrac{12}{x-1}$ is concave upward and those on which it is concave downward. Find the points of inflection.

(57) Find all relative extrema using the second derivative test of $f(x) = \frac{3}{4}\,x^4 + 5x^3 + 9x^2 - \frac{15}{4}$

(58) Find all relative extrema using the second derivative test of $f(x) = \dfrac{x^3 - 54}{x}$

(59) Find all relative extrema using the second derivative test of $f(x) = \dfrac{-3\,\left(x^2 + 2x + 4\right)}{x+2}$

(60) Use the derivative method to sketch the graph of
$f(x) = \dfrac{(x-2)(3x+1)}{(x-1)^2}$ with $f'(x) = \dfrac{9-x}{(x-1)^3}$ and $f''(x) = \dfrac{2\,(x-13)}{(x-1)^4}$

(61) Use the derivative method to sketch the graph of
$f(x) = \left(\dfrac{x+3}{x+1}\right)^2$ with $f'(x) = \dfrac{-4(x+3)}{(x+1)^3}$ and $f''(x) = \dfrac{8\,(x+4)}{(x+1)^4}$

(62) Use the derivative method to sketch the graph of
$f(x) = \dfrac{2x^2}{x^2-1}$ with $f'(x) = \dfrac{-4x}{(x^2-1)^2}$ and $f''(x) = \dfrac{4\,(3x^2+1)}{(x^2-1)^3}$

(63) Use the derivative method to sketch the graph of
$f(x) = \dfrac{4}{x^2-4x}$ with $f'(x) = \dfrac{8(2-x)}{(x^2-4x)^2}$ and $f''(x) = \dfrac{8\,(3x^2-12x+16)}{(x^2-4x)^3}$

(64) Use the derivative method to sketch the graph of
$f(x) = \frac{1}{5}\,x^5 - \frac{8}{3}\,x^3 - 9x$ with $f'(x) = \left(x^2-9\right)\left(x^2+1\right)$ and $f''(x) = 4x\,\left(x^2-4\right)$

(65) Use the derivative method to sketch the graph of
$f(x) = -x^3 + 18x^2 + 200$ with $f'(x) = 3x(12-x)$ and $f''(x) = 36 - 6x$

(66) Use the derivative method to sketch the graph of
$f(x) = (x+2)^4\,(4-3x)$ with $f'(x) = 5\,(x+2)^3\,(2-3x)$ and $f''(x) = -60x\,(x+2)^2$

(67) Use the derivative method to sketch the graph of
$f(x) = 3x + \dfrac{3}{x+1}$ with $f'(x) = \dfrac{3x(x+2)}{(x+1)^2}$ and $f''(x) = \dfrac{6}{(x+1)^3}$

(68) Use the derivative method to sketch the graph of
$f(x) = \dfrac{x^2}{x+2}$ with $f'(x) = \dfrac{x(x+4)}{(x+2)^2}$ and $f''(x) = \dfrac{8}{(x+2)^3}$

(69) A cherry plantation contains 70 trees per acre. The average yield is 600 cherries per tree. The farmer wishes to plant more trees to maximize the yield per acre. From past evidence the farmer has determined that for each additional tree planted per acre, the yield per tree is reduced by 3 cherries.
Find the number of trees per acre that will maximize the yield per acre.

(70) A Heating Company has 20 stores on the South Shore of Montreal, each of which has an average income of \$24 000 per week. After study, the company notices that for each new store opened, the average weekly income drops \$800. How many new stores should be opened to maximize income?

(71) The average cost in dollars/unit to manufacture a product is $\overline{C} = \frac{1}{3}x^2 + 18x - 1440 + \frac{20\,000}{x}$.
How many units should be made in order to minimize the cost and what is the minimum cost?

(72) The selling price in dollars/unit to sell a product is $p = -\frac{1}{3}x^2 + 2x + 96$.
How many units should be made in order to maximize the revenue and what is the maximum revenue?

(73) A company manufactures and sells x units of a product per week. If the weekly average cost in dollars per unit is $\overline{C} = \frac{4}{3}x^2 + 5x + 34$ and the selling price in dollars per unit is $p = x^2 + 20x + 250$. Find for each week, the maximum revenue, the maximum profit, the production level that will realize the maximum profit and the price that the company should charge for each unit.

(74) If the average manufacturing cost (in dollars per unit) of a product is given by $\overline{C} = 3x^3 + x^2 + 4x$ where x is the number of units manufactured and the selling price in dollars per unit is $p = 2x^3 + 25x^2 + 130x$, what is the production to maximize the profit? What is the maximum profit?

(75) The demand function for a product is given by $p = \dfrac{60x}{x - 10}$ where p is the price per unit when x units are demanded.
(a) Determine the intervals of elasticity of demand
(b) If the price of the product at \$80 increases by 2%, what is the approximate percentage in demand?
(c) Will the total revenue increase, decrease or remain constant at $p = \$80$?

(76) The demand function for a product is given by $p = \sqrt{300 - x}$ where p is the price per unit when x units are demanded.
(a) Determine the intervals of elasticity of demand
(b) If the price of the product at \$15 increases by 1%, what is the approximate percentage in demand?
(c) Will the total revenue increase, decrease or remain constant at $p = \$15$?

(77) The demand function for a product is given by $p = -x^2 + 15x + 225$ where p is the price per unit when x units are demanded.
(a) Determine the intervals of elasticity of demand
(b) If the price of the product at \$125 decreases by 4%, what is the approximate percentage in demand?
(c) Will the total revenue increase, decrease or remain constant at $p = \$125$?

(78) A land of 4200 square meters will be subdivided into 2 equal rectangular fields with the inner fence costs \$10 per linear meter and the outer fence costs \$30 per linear meter. Find the minimum cost of the fence.

(79) A farmer has \$3600 to fence a land next to a river. She wants to enclose in a rectangular field and the fence opposite to the river costs \$15 per linear meter and the fence of the other two sides costs \$25 per linear meter. Find the maximum area of the land.

(80) The demand function for a product is given by $p = \dfrac{x^2+400}{x^2}$ where p is the price per unit when x units are demanded.
(a) Determine the intervals of elasticity of demand
(b) If the price of the product at \$5 increases by 8%, what is the approximate percentage in demand?
(c) Will the total revenue increase, decrease or remain constant at $p = \$5$?

(81) Sketch $f(x)$ with the following requirements: points at $(-3,2)$, $(-2,0)$, $(0,-2)$, $(1,0)$, $\lim_{x\to+\infty} f(x) = 1$
at $x < -3$: $f'(x) < 0$; $f''(x) < 0$
at $-3 < x < 0$: $f'(x) < 0$; $f''(x) > 0$
at $x > 0$: $f'(x) > 0$; $f''(x) < 0$

(82) Sketch $f(x)$ with the following requirements:
points at $(-3,0)$, $(-2,1)$, $(-1,0)$, $(0,-0.5)$, $(1,-2)$, $\lim_{x\to+\infty} f(x) = 0$
$f'(x) < 0$ at $-2 < x < 1$; $f'(x) > 0$ at $x < -2$ or $x > 1$
$f''(x) < 0$ at $x < -2$ or $x > -1$; $f''(x) > 0$ at $-2 < x < -1$

(83) Sketch $f(x)$ with the following requirements: points at $(-2,0)$, $(-1,-1)$, $(0,0)$
vertical asymptote at $x = 1$ and $\lim_{x\to+\infty} f(x) = 2$
at $x < -1$: $f'(x) < 0$; $f''(x) < 0$
at $-1 < x < 0$: $f'(x) > 0$; $f''(x) < 0$
at $0 < x < 1$: $f'(x) > 0$; $f''(x) > 0$
at $x > 1$: $f'(x) < 0$; $f''(x) > 0$

(84) Sketch $f(x)$ with the following requirements: points at $(-3,0)$, $(-1,-1)$, $(0,-2)$, $(1,-1)$
vertical asymptote at $x = -2$; $\lim_{x\to-\infty} f(x) = 1$; $\lim_{x\to+\infty} f(x) = 0$
$f'(x) < 0$ at $x < -2$ or $-1 < x < 0$; $f'(x) > 0$ at $-2 < x < -1$ or $0 < x < 1$ or $x > 1$
$f''(x) < 0$ at $x < -2$ or $-2 < x < -1$ or $x > 1$; $f''(x) > 0$ at $-1 < x < 1$

(85) Sketch $f(x)$ with the following requirements: points at $(-2,2)$, $(0,1)$, $(2,2)$ and $\lim_{x\to-\infty} f(x) = 0$
at $x < -2$: $f'(x) > 0$; $f''(x) > 0$
at $-2 < x < 0$: $f'(x) < 0$; $f''(x) < 0$
at $0 < x < 2$: $f'(x) > 0$; $f''(x) < 0$
at $x > 2$: $f'(x) > 0$; $f''(x) > 0$

(86) Sketch $f(x)$ with the following requirements: points at $(-2,1)$, $(0,-1)$, $(2,0)$ and $\lim_{x\to+\infty} f(x) = 2$
$f'(x) < 0$ at $x < 0$; $f'(x) > 0$ at $x > 0$
$f''(x) < 0$ at $-2 < x < 0$ or $x > 2$; $f''(x) > 0$ at $x < -2$ or $0 < x < 2$

(87) Sketch $f(x)$ with the following requirements: points at $(-2,0)$, $(0,0)$
vertical asymptote at $x = -1$ and $\lim_{x\to+\infty} f(x) = 1$
at $x < -2$: $f'(x) > 0$; $f''(x) < 0$
at $-2 < x < -1$: $f'(x) < 0$; $f''(x) < 0$
at $-1 < x < 0$: $f'(x) < 0$; $f''(x) > 0$
at $x > 0$: $f'(x) > 0$; $f''(x) < 0$

(88) Sketch $f(x)$ with the following requirements: Domain: $-3 < x \leq 4$; points at $(-1,0)$, $(0,-1)$, $(1,0)$, $(4,2)$
$f'(x) < 0$ at $-3 < x < 0$; $f'(x) > 0$ at $0 < x < 4$
$f''(x) < 0$ at $-3 < x < -1$ or $1 < x < 4$; $f''(x) > 0$ at $-1 < x < 1$

(89) Sketch $f(x)$ with the following requirements: Domain: $-2 \leq x < 4$; points at $(-2,-1)$, $(0,0)$, $(2,2)$
$f'(x) < 0$ at $2 < x < 4$; $f'(x) > 0$ at $-2 < x < 2$
$f''(x) < 0$ at $-2 < x < 0$ or $2 < x < 4$; $f''(x) > 0$ at $0 < x < 2$

(90) Sketch $f(x)$ with the following requirements: Domain: $-4 < x \leq 3$; points at $(0,1)$, $(1,0)$, $(3,2)$
vertical asymptote at $x = -2$
at $-4 < x < -2$: $f'(x) > 0$; $f''(x) > 0$
at $-2 < x < 0$: $f'(x) < 0$; $f''(x) > 0$
at $0 < x < 1$: $f'(x) < 0$; $f''(x) < 0$
at $1 < x < 3$: $f'(x) > 0$; $f''(x) < 0$

(91) The demand function for a product is given by $p = \dfrac{x^2}{x^2+1}$ where p is the price per unit when x units are demanded.
(a) Find the elasticity of demand
(b) Evaluate the elasticity of demand at 25 units
(c) Interpret the result in (b)
(d) If the price of the product at \$1 increases by 1%, what is the approximate percentage in demand?

(92) Find the absolute extrema of the function $f(x) = \frac{x^2+25}{4x}$ on the interval $[2,6]$

(93) Find all relative extrema using the second derivative test of $f(x) = \frac{1}{4}x^4 + x^3 - \frac{1}{2}x^2 - 3x$

(94) Find all critical points of the function $f(x) = \dfrac{(x-1)^2}{x^2-4}$

(95) Find the intervals on which the graph of $f(x) = x^4 + 2x^3 - 12x^2$ is concave upward and those on which it is concave downward. Find the points of inflection.

(96) Find the intervals on which the graph of $f(x) = \dfrac{x^3}{x+2}$ is increasing and those on which it is decreasing. Find the critical points.

(97) A company manufactures and sells x units of a product per day. If the daily cost in dollars is $C = 3x^3 + x^2 + 20x$ and the revenue in dollars is $R = x^3 + 40x^2 + 200x + 275$.
Find the maximum profit.

(98) Find the critical point(s) and the inflection point(s) of the function $f(x) = 4x^5 + 5x^4 - 80x^3$

(99) Find the absolute extrema of the function $f(x) = x^4 - 8x^2$ on the interval $[-2,3]$

(100) The demand function for a product is given by $p = \sqrt{100 - x^2}$ where p is the price per unit when x units are demanded.
(a) Find the elasticity of demand
(b) Evaluate the elasticity of demand at 6 dollars
(c) Interpret the result in (b)
(d) If the price of the product at \$6 decreases by 10%, what is the approximate percentage in demand?