Math 201-103-RE

Practice Assignment 5

Applications of the Derivative

Answers:

(1) The critical numbers are: $x = -1$ and $x = 3$, $f(x)$ is increasing at $-1 < x < 3$
$f(x)$ is decreasing at $x < -1$ or $x > 3$

(2) The critical numbers are: $x = -1$, $x = 0$ and $x = 1$, $f(x)$ is increasing at $x < -1$ or $x > 1$
$f(x)$ is decreasing at $-1 < x < 0$ or $0 < x < 1$

(3) The critical numbers are: $x = -2$, $x = 0$ and $x = 2$, $f(x)$ is increasing at $-2 < x < 0$ or $0 < x < 2$;
$f(x)$ is decreasing at $x < -2$ or $x > 2$

(4) The critical numbers are: $x = -2$, $x = 0$ and $x = 1$, $f(x)$ is increasing at $-2 < x < 0$ or $x > 1$
$f(x)$ is decreasing at $x < -2$ or $0 < x < 1$

(5) The critical numbers are: $x = -3$, $x = 0$ and $x = 3$, $f(x)$ is increasing at $x < -3$ or $x > 3$;
$f(x)$ is decreasing at $-3 < x < 0$ or $0 < x < 3$

(6) relative maximum at $(5, -8)$ and relative minimum at $(-1, 4)$

(7) relative minimum at $(-3, -11.5)$

(8) relative maximum at $(-2, -7)$ and relative minimum at $(0, 9)$

(9) relative minimum at $(-1, 6)$

(10) Absolute maximum is 9.5 at $x = 3$ and absolute minimum is -3 at $x = 2$

(11) Absolute maximum is -3 at $x = 2$ and absolute minimum is -5 at $x = 1$

(12) Absolute maximum is 28.5 at $x = 3$ and absolute minimum is -7.33 at $x = 2$

(13) Absolute maximum is 17.5 at $x = 3$ and absolute minimum is -4 at $x = 2$

(14) $f(x)$ is concave up at $x < 0$ or $x > 2$ and concave down at $0 < x < 2$. P.I. at $(2\ ,\ 5)$

(15) $f(x)$ is concave up at $0 < x < 1$ and concave down at $x < 0$ or $x > 1$. P.I. at $(0\ ,\ 0)$ and $(1\ ,\ 6)$

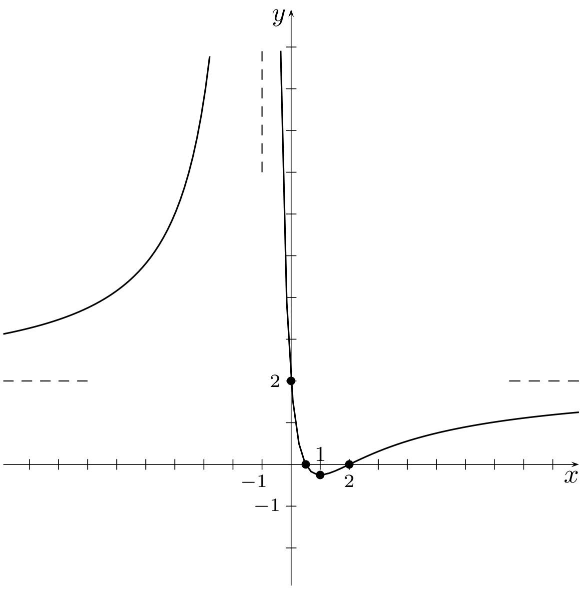
(16) $f(x)$ is concave up at $1 < x < 3$ or $x > 3$ and concave down at $x < 1$. P.I. at $(3\ ,\ 5)$

(17) Relative maximum at $\left(1, \frac{49}{4}\right)$, relative minimum at $(\ 4\ ,\ 1)$ and relative minimum at $\left(0, \frac{35}{3}\right)$.

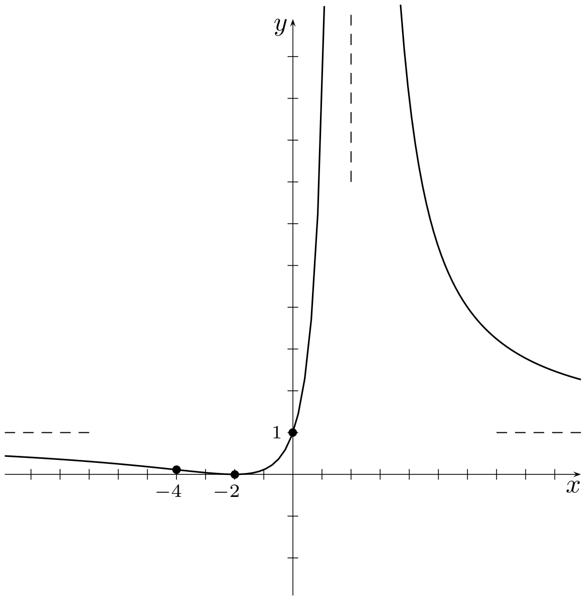
(18) Relative minimum at $(2,\ 72)$

(19) Relative maximum at $(-3, -50)$ and relative minimum at $(1,\ 30)$

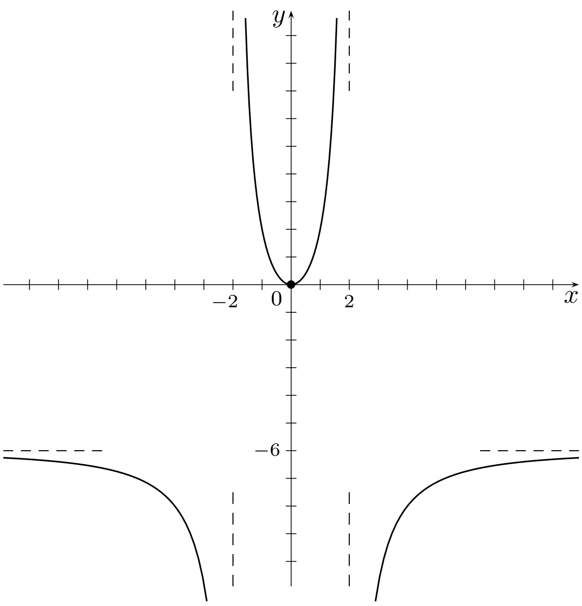
(20) $f(x) = \dfrac{(x-2)(2x-1)}{(x+1)^2}$



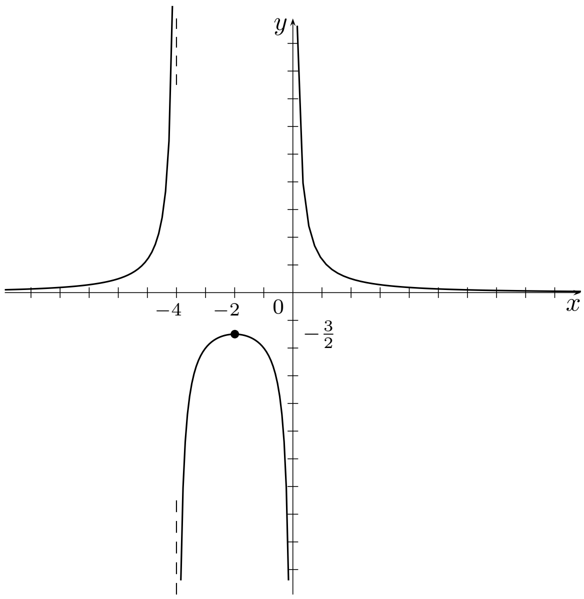
(21) $f(x) = \left(\dfrac{x+2}{x-2}\right)^2$



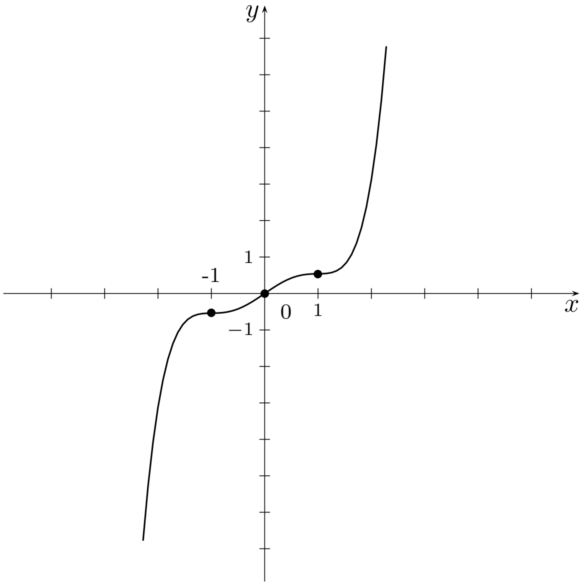
(22) $f(x) = \dfrac{6x^2}{4-x^2}$



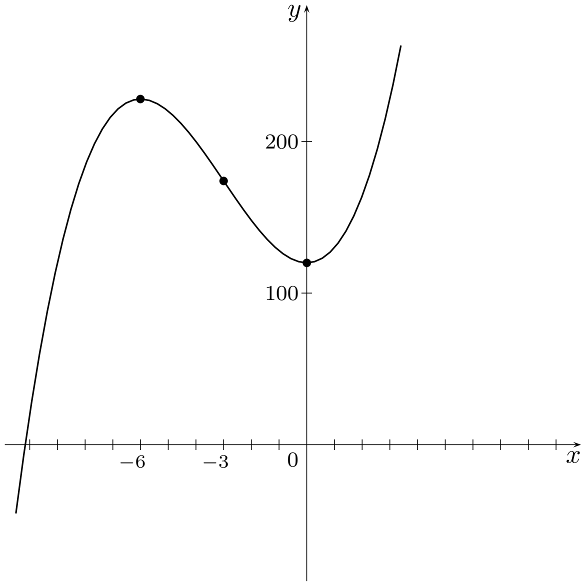
(23) $f(x) = \dfrac{6}{x^2+4x}$



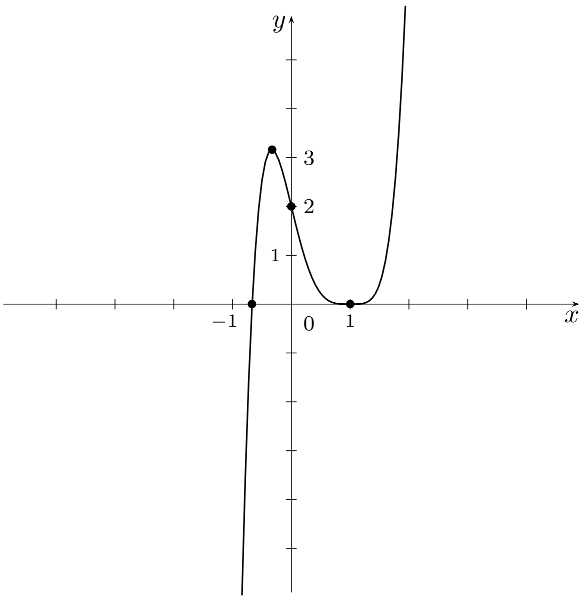
(24) $f(x) = \frac{1}{5}x^5 - \frac{2}{3}x^3 + x$



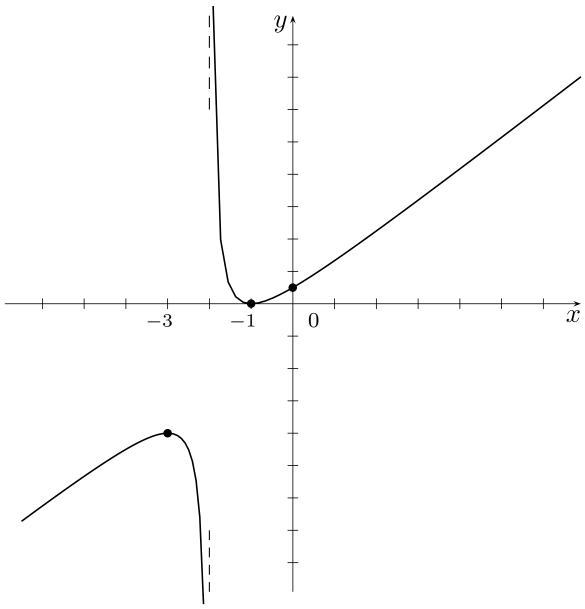
(25) $f(x) = x^3 + 9x^2 + 120$



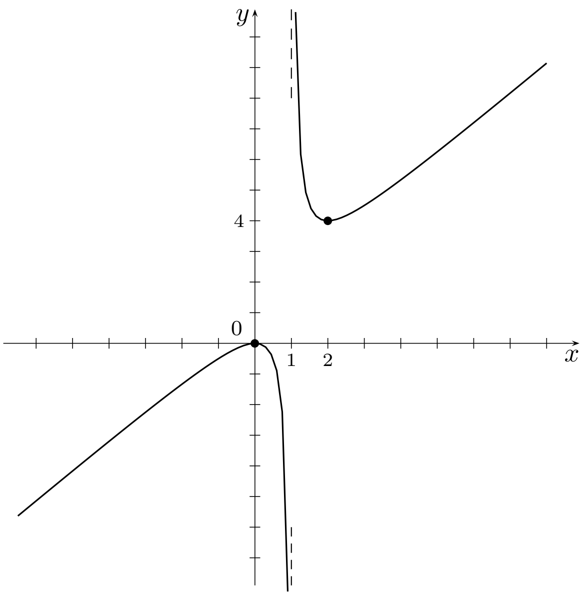
(26) $f(x) = (x-1)^4(3x+2)$



(27) $f(x) = x + \dfrac{1}{x+2}$



(28) $f(x) = \dfrac{x^2}{x-1}$



(29) 95 trees per acre will maximize the yield per acre.
(30) 40 new stores should be opened to maximize income.
(31) 40 units should minimize the cost and the minimum cost is \$18 500
(32) 27 units should maximize the revenue and the maximum revenue is \$6925.50
(33) the maximum revenue is \$42 000 ; the maximum profit is \$7 200 ; 30 units will realize the maximum profit and the price per unit is \$1400.
(34) 18 units should be manufactured to maximize the profit ; the maximum profit is \$11 664.

(35) (a) elastic at $x > 24$, inelastic at $12 < x < 24$ and unit elasticity at $x = 24$
(b) the approximate percentage in demand will increase by 1% (c) explain your answer

(36) (a) elastic at $0 < x < \frac{800}{3}$, inelastic at $x < \frac{800}{3}$ and unit elasticity at $x = \frac{800}{3}$
(b) the approximate percentage in demand will increase by 2% (c) explain your answer

(37) (a) elastic at $0 < x < 40$, inelastic at $40 < x < 63.8$ and unit elasticity at $x = 40$
(b) the approximate percentage in demand will increase by 11% (c) explain your answer

(38) The minimum cost is \$6000 (39) The dimensions are 40 m by 160 m

(40) (a) elastic at $x > 0$
(b) the approximate percentage in demand will increase by 10% (c) explain your answer

(41) The critical numbers are: $x = -3$ and $x = 2$, $f(x)$ is increasing at $-3 < x < 2$
$f(x)$ is decreasing at $x < -3$ or $x > 2$

(42) The critical numbers are: $x = -2$, $x = 0$ and $x = 2$, $f(x)$ is increasing at $-2 < x < 0$ or $0 < x < 2$
$f(x)$ is decreasing at $x < -2$ or $x > 2$

(43) The critical numbers are: $x = -8$ and $x = 0$, $f(x)$ is increasing at $-8 < x < 0$
$f(x)$ is decreasing at $x < -8$ or $x > 0$

(44) The critical numbers are: $x = 0$, $x = 1$ and $x = 4$, $f(x)$ is increasing at $0 < x < 1$ or $x > 4$
$f(x)$ is decreasing at $x < 0$ or $1 < x < 4$

(45) The critical numbers are: $x = -2$, $x = 0$ and $x = 2$, $f(x)$ is increasing at $-2 < x < 0$ or $0 < x < 2$;
$f(x)$ is decreasing at $x < -2$ or $x > 2$

(46) relative maximum at $(-1, -3)$ and relative minimum at $(3,\ 5)$

(47) relative minimum at $(3, -16.25)$

(48) relative maximum at $(-3,-5)$ and relative minimum at $(-1,3)$

(49) relative minimum at (1, 9)

(50) Absolute maximum is -14 at $x=-1$ and absolute minimum is -142 at $x=-3$

(51) Absolute maximum is 5.54 at $x=5$ and absolute minimum is 4.5 at $x=3$

(52) Absolute maximum is 426.67 at $x=5$ and absolute minimum is -5.33 at $x=-1$

(53) Absolute maximum is 41.6 at $x=-2$ and absolute minimum is 7.7 at $x=1$

(54) $f(x)$ is concave up at $x<0$ or $x>3$ and concave down at $0<x<3$. P.I. at (3 , 4)

(55) $f(x)$ is concave up at $x<-2$ or $x>0$ and concave down at $-2<x<0$. P.I. at (0 , 0) and $(-2,-6)$

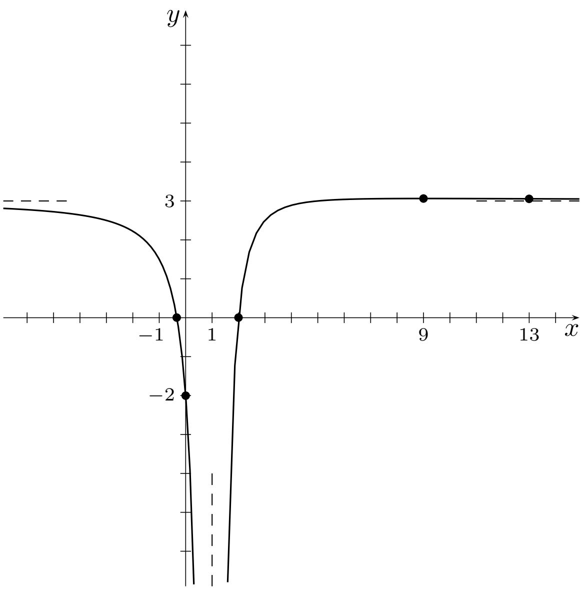
(56) $f(x)$ is concave up at $x<-1$ or $x>1$ and concave down at $-1<x<1$. P.I. at $(-1,-4.5)$

(57) Relative maximum at $\left(-2,\frac{17}{4}\right)$, relative minimum at $(-3,3)$ and relative minimum at $\left(0,-\frac{15}{4}\right)$.

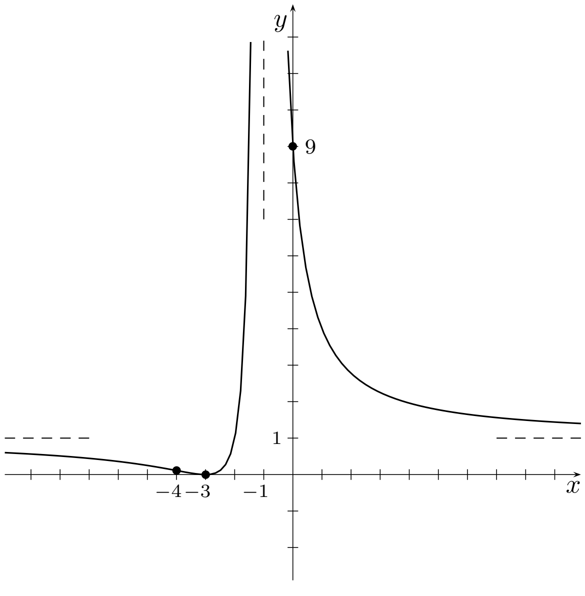
(58) Relative minimum at $(-3,27)$

(59) Relative maximum at $(0,-6)$ and relative minimum at $(-4,18)$

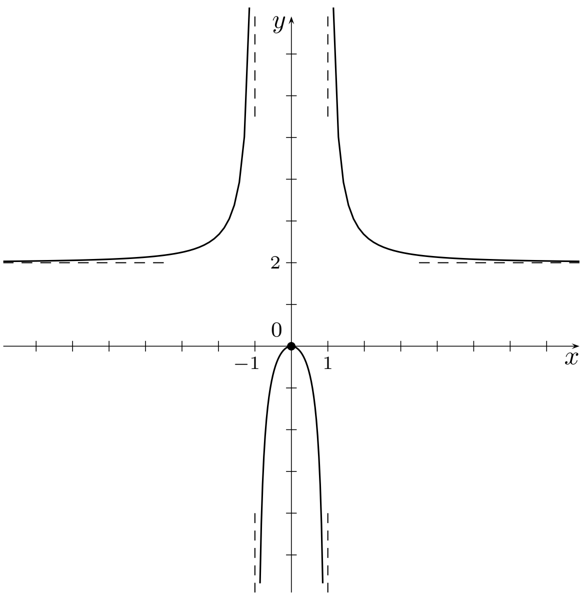
(60) $f(x)=\dfrac{(x-2)(3x+1)}{(x-1)^2}$



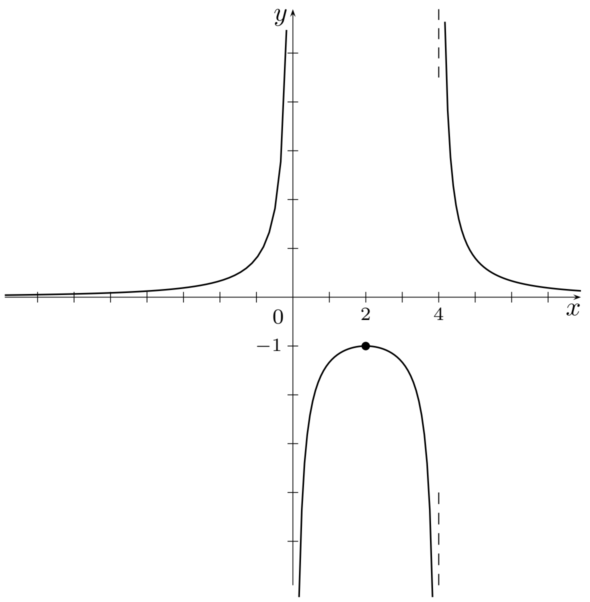
(61) $f(x)=\left(\dfrac{x+3}{x+1}\right)^2$



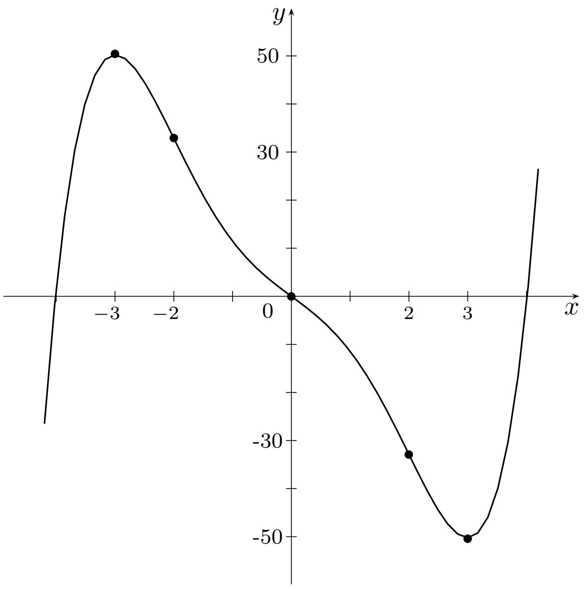
(62) $f(x) = \dfrac{2x^2}{x^2 - 1}$



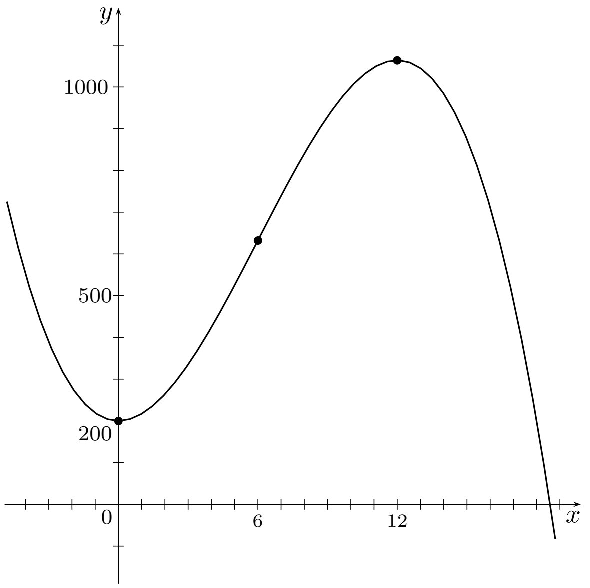
(63) $f(x) = \dfrac{4}{x^2 - 4x}$



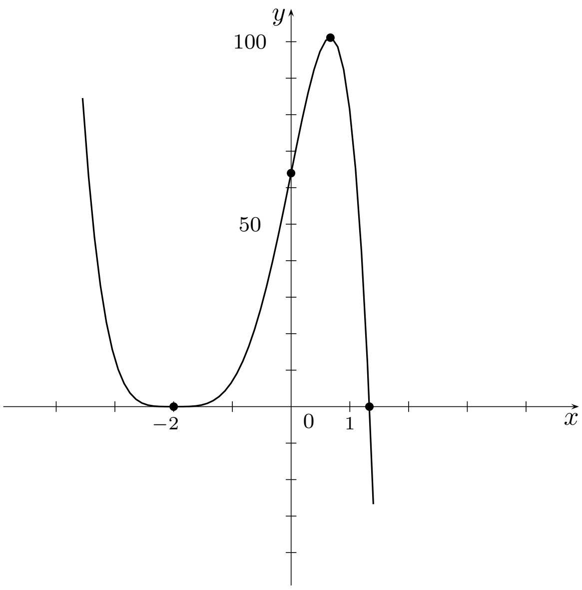
(64) $f(x) = \frac{1}{5}\,x^5 - \frac{8}{3}\,x^3 - 9x$



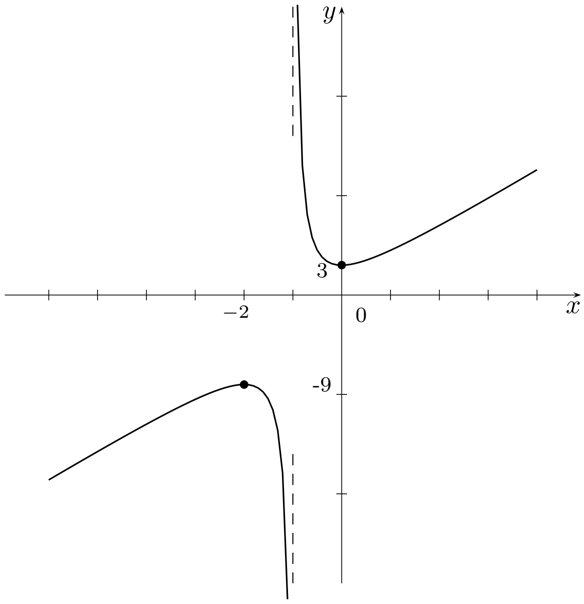
(65) $f(x) = -x^3 + 18x^2 + 200$



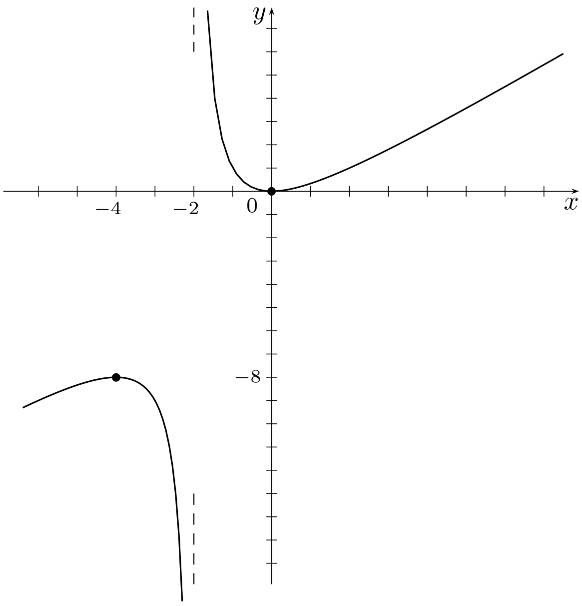
(66) $f(x) = (x+2)^4\,(4-3x)$



(67) $f(x) = 3x + \dfrac{3}{x+1}$



(68) $f(x) = \dfrac{x^2}{x+2}$



(69) 135 trees per acre will maximize the yield per acre.
(70) 5 new stores should be opened to maximize income.
(71) 24 units should minimize the cost and the minimum cost is \$416
(72) 12 units should maximize the revenue and the maximum revenue is \$864
(73) the maximum revenue is \$81 576 ; the maximum profit is \$11 664 ; 36 units will realize the maximum profit and the price per unit is \$2266.
(74) 21 units should be manufactured to maximize the profit ; the maximum profit is \$83 349.

(75) (a) elastic at $x > 20$, inelastic at $10 < x < 20$ and unit elasticity at $x = 20$
(b) the approximate percentage in demand decreases by 6% (c) explain your answer

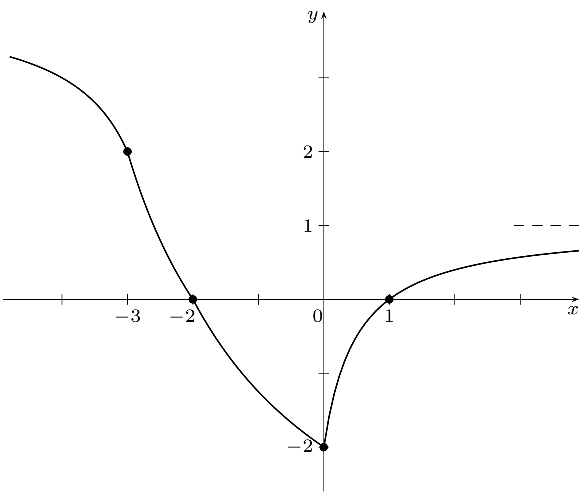
(76) (a) elastic at $0 < x < 200$, inelastic at $200 < x < 300$ and unit elasticity at $x = 200$
(b) the approximate percentage in demand will decrease by 6% (c) explain your answer

(77) (a) elastic at $0 < x < 15$, inelastic at $15 < x < 24$ and unit elasticity at $x = 15$
(b) the approximate percentage in demand will increase by 1% (c) explain your answer

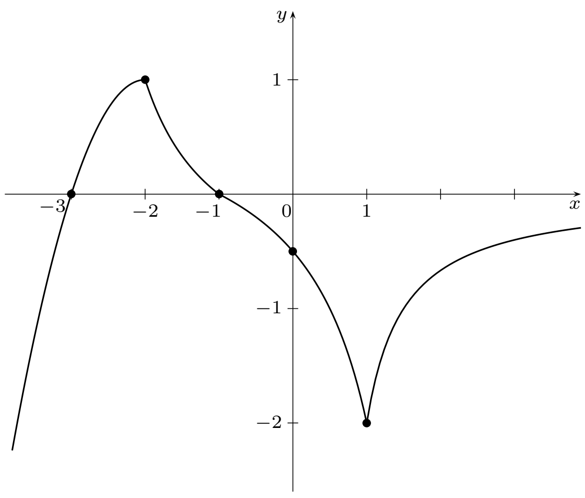
(78) The minimum cost is \$8400 (79) The maximum area is 4320 square meters

(80) (a) elastic at $x > 20$, inelastic at $0 < x < 20$ and unit elasticity at $x = 20$
(b) the approximate percentage in demand will decrease by 5% (c) explain your answer

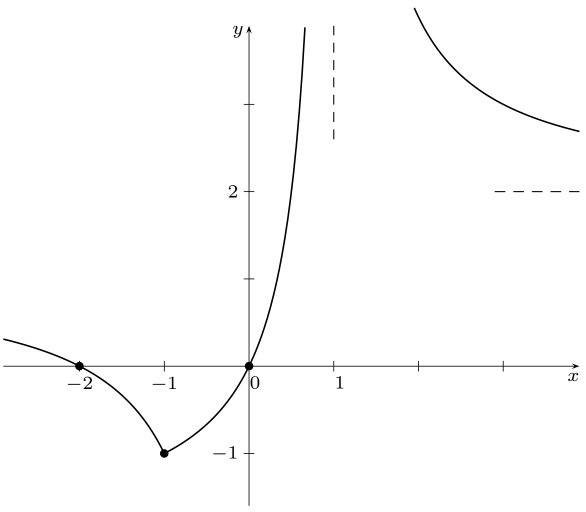
(81)



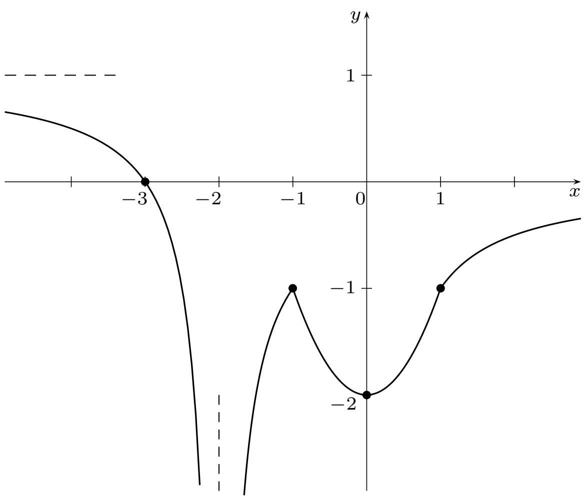
(82)



(83)



(84)



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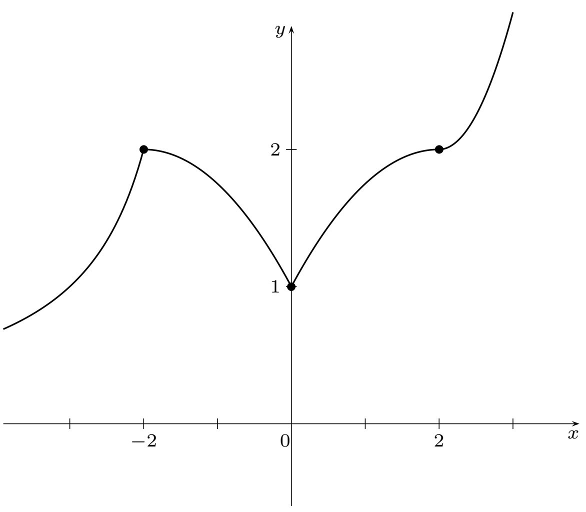

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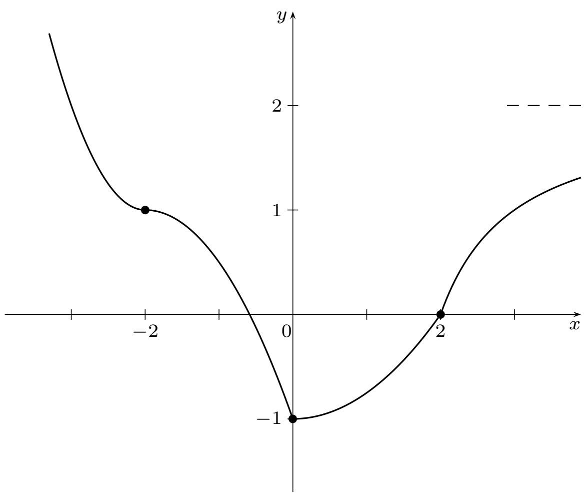
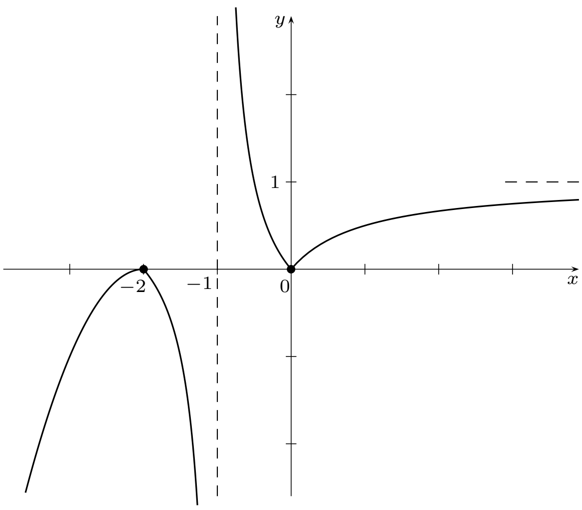
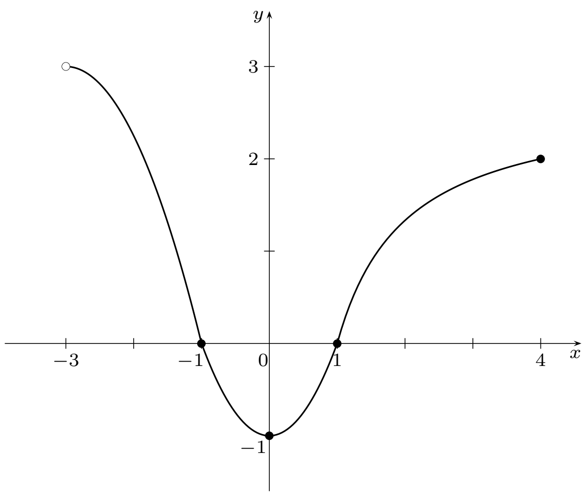


Math 201-103-RE Practice Assignment 5 Answers:

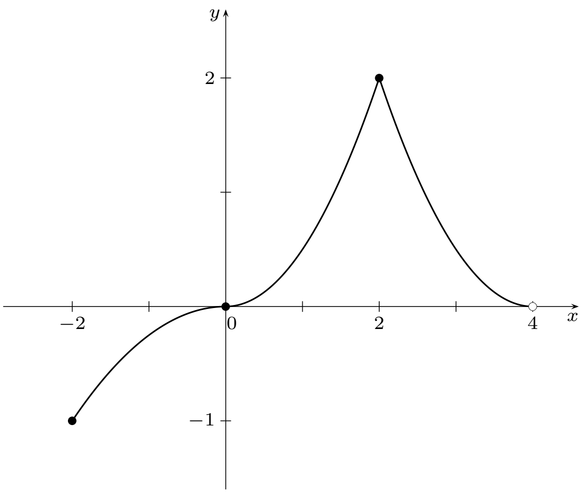
(87)



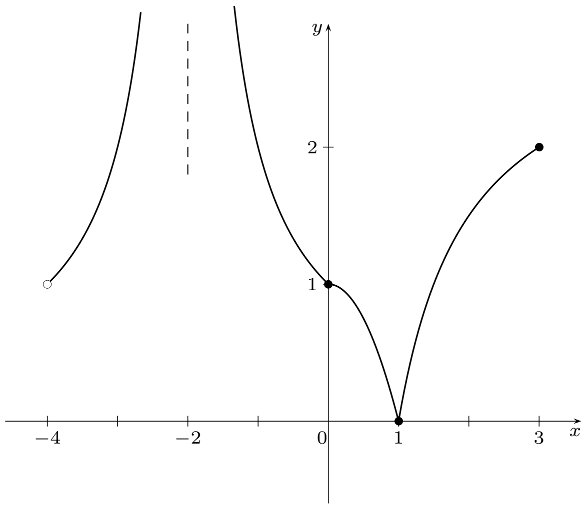
(88)



(89)



(90)



(91 a) $\eta = \frac{x^2+1}{2}$; (91 b) 313 ; (91 c) elastic ; (91 d) +313%

(92) absolute maximum is $\frac{29}{8} = 3.625$ at $x = 2$ and absolute minimum is $\frac{5}{2} = 2.5$ at $x = 5$

(93) relative maximum at $\left(-1, \frac{7}{4}\right)$; relative minimum at $\left(-3, -\frac{9}{4}\right)$ and relative minimum at $\left(1, -\frac{9}{4}\right)$

(94) relative maximum at $(1, 0)$ and relative minimum at $\left(4, \frac{3}{4}\right)$

(95) concave up at $x < -2$ or $x > 1$; concave down at $-2 < x < 1$ and points of inflection at $(1, -9), (-2, -48)$

(96) increasing at $-3 < x < -2$ or $x > -2$; decreasing at $x < -3$ and relative maximum at $(-3, 27)$
and relative minimum at $(0, 0)$

(97) maximum profit of \$5000

(98) relative maximum at $(-3, 1593)$; relative minimum at $(2, -432)$; point of inflection at $(0, 0)$

(99) absolute maximum is 9 at $x = 3$ and absolute minimum is -16 at $x = -2$ and $x = 2$

(100 a) $\eta = \frac{x^2-100}{x^2}$; (100 b) -0.5625 ; (100 c) inelastic ; (100 d) +5.625%