Here is the integral table used:

1. $$\int \frac{u\,du}{a+bu} = \frac{u}{b} - \frac{a}{b^2}\ln|a+bu| + C$$

2. $$\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$$

3. $$\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$$

4. $$\int \frac{u\,du}{(a+bu)^2} = \frac{1}{b^2}\left(\ln|a+bu| + \frac{a}{a+bu}\right) + C$$

5. $$\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} + \frac{1}{a^2}\ln\left|\frac{u}{a+bu}\right| + C$$

6. $$\int \frac{du}{u^2(a+bu)^2} = -\frac{a+2bu}{a^2u(a+bu)} + \frac{2b}{a^3}\ln\left|\frac{a+bu}{u}\right| + C$$

7. $$\int \frac{u^2\,du}{(a+bu)^2} = \frac{1}{b^3}\left(bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right) + C$$

8. $$\int \frac{u^2\,du}{(a+bu)^3} = \frac{1}{b^3}\left(\frac{2a}{a+bu} - \frac{a^2}{2(a+bu)^2} + \ln|a+bu|\right) + C$$

9. $$\int \frac{u^2\,du}{(a+bu)^n} \qquad (n \neq 1,2,3)$$
$$= \frac{1}{b^3}\left(\frac{-1}{(n-3)(a+bu)^{n-3}} + \frac{2a}{(n-2)(a+bu)^{n-2}} - \frac{a^2}{(n-1)(a+bu)^{n-1}}\right) + C$$

10. $$\int \frac{du}{(a+bu)(c+ku)} = \frac{1}{bc-ak}\ln\left|\frac{a+bu}{c+ku}\right| + C \qquad (bc-ak\neq 0)$$

11. $$\int \frac{u\,du}{(a+bu)(c+ku)} = \frac{1}{bc-ak}\left(\frac{c}{k}\ln|c+ku| - \frac{a}{b}\ln|a+bu|\right) + C \qquad (bc-ak\neq 0)$$

12. $$\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C \qquad (a>0)$$

13. $$\int u\sqrt{a+bu}\,du = \frac{2}{15b^2}(3bu-2a)(a+bu)^{3/2} + C$$

14. $$\int u^2\sqrt{a+bu}\,du = \frac{2}{105b^3}(8a^2-12abu+15b^2u^2)(a+bu)^{3/2} + C$$

15. $$\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu} + C$$

16. $$\int \frac{u^2\,du}{\sqrt{a+bu}} = \frac{2}{15b^3}(3b^2u^2-4abu+8a^2)\sqrt{a+bu} + C$$

17. $$\int \frac{du}{u\sqrt{a+bu}} = \frac{1}{\sqrt{a}} \ln\left|\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + C \qquad (a>0)$$

18. $$\int \frac{\sqrt{a+bu}\,du}{u} = 2\sqrt{a+bu} + \sqrt{a}\ln\left|\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + C \qquad (a>0)$$

19. $$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C \qquad (a>0)$$

20. $$\int \frac{du}{u^2\sqrt{a^2-u^2}} = -\frac{\sqrt{a^2-u^2}}{a^2u} + C$$

21. $$\int \frac{\sqrt{a^2-u^2}\,du}{u} = \sqrt{a^2-u^2} - a\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C \qquad (a>0)$$

22. $$\int \sqrt{u^2\pm a^2}\,du = \frac{1}{2}\left(u\sqrt{u^2\pm a^2} \pm a^2\ln\left|u+\sqrt{u^2\pm a^2}\right|\right) + C$$

23. $$\int u^2\sqrt{u^2\pm a^2}\,du = \frac{u}{8}\left(2u^2\pm a^2\right)\sqrt{u^2\pm a^2} - \frac{a^4}{8}\ln\left|u+\sqrt{u^2\pm a^2}\right| + C$$

24. $$\int \frac{\sqrt{u^2+a^2}\,du}{u} = \sqrt{u^2+a^2} - a\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right| + C \qquad (a>0)$$

25. $$\int \frac{\sqrt{u^2\pm a^2}\,du}{u^2} = -\frac{\sqrt{u^2\pm a^2}}{u} + \ln\left|u+\sqrt{u^2\pm a^2}\right| + C$$

26. $$\int \frac{du}{\sqrt{u^2\pm a^2}} = \ln\left|u+\sqrt{u^2\pm a^2}\right| + C$$

27. $$\int \frac{du}{u\sqrt{u^2+a^2}} = \frac{1}{a}\ln\left|\frac{\sqrt{u^2+a^2}-a}{u}\right| + C \qquad (a>0)$$

28. $$\int \frac{u^2\,du}{\sqrt{u^2\pm a^2}} = \frac{1}{2}\left(u\sqrt{u^2\pm a^2} \mp a^2\ln\left|u+\sqrt{u^2\pm a^2}\right|\right) + C$$

29. $$\int \frac{du}{u^2\sqrt{u^2\pm a^2}} = \mp\frac{\sqrt{u^2\pm a^2}}{a^2u} + C$$

30. $$\int u^n e^{au}\,du = \frac{u^n e^{au}}{a} - \frac{n}{a}\int u^{n-1}e^{au}\,du \qquad (n\geq 1)$$

31. $$\int u^n \ln^m u\,du = \frac{u^{n+1}}{n+1}\ln^m u - \frac{m}{n+1}\int u^n \ln^{m-1} u\,du \quad (m, n\neq -1)$$

32. $$\int (\ln u)^n\,du = u\,(\ln u)^n - n\int (\ln u)^{n-1}\,du$$

33. $\displaystyle\int \frac{du}{a+be^{cu}} = \frac{1}{ac}\left(cu - \ln\left|a+be^{cu}\right|\right) + C$

34. $\displaystyle\int \frac{du}{(a^2-u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2-u^2}} + C$

35. $\displaystyle\int (u^2 \pm a^2)^{3/2} du = \frac{u}{8}\left(2u^2 \pm 5a^2\right)\sqrt{u^2 \pm a^2} + \frac{3a^4}{8}\ln\left|u+\sqrt{u^2 \pm a^2}\right| + C$

36. $\displaystyle\int \frac{du}{(u^2 \pm a^2)^{3/2}} = \frac{\pm u}{a^2\sqrt{u^2 \pm a^2}} + C$

37. $\displaystyle\int \frac{u^2\,du}{(u^2 \pm a^2)^{3/2}} = \frac{-u}{\sqrt{u^2 \pm a^2}} + \ln\left|u+\sqrt{u^2 \pm a^2}\right| + C$

38. $\displaystyle\int \sqrt{\frac{a+u}{b+u}}\,du = \sqrt{(a+u)(b+u)} + (a-b)\ \ln\left(\sqrt{a+u}+\sqrt{b+u}\right) + C$

39. $\displaystyle\int \frac{du}{\sqrt{(a+u)(b+u)}} = \ln\left|\frac{a+b}{2} + u + \sqrt{(a+u)(b+u)}\right| + C$

40. $\displaystyle\int \sqrt{a+bu+cu^2}\,du = \frac{2cu+b}{4c}\sqrt{a+bu+cu^2} - \frac{b^2-4ac}{8c^{3/2}}\ \ln\left|2cu+b+2\sqrt{c}\sqrt{a+bu+cu^2}\right| + C$
$(c > 0)$

Example 1:

Use the table to determine $\displaystyle\int \frac{30x^5}{(4+5x^3)^2}\,dx = \int \frac{x^3}{(4+5x^3)^2}\,30x^2\,dx$

substitute: $u = x^3\,,\ du = 3x^2\,dx\,,\ 10\,du = 30x^2\,dx\,,\ a=4\,,\ b=5 \implies \displaystyle\int \frac{u}{(a+bu)^2}\,10\,du \implies 10$ (answer F4)

Use F4: $\displaystyle\int \frac{u\,du}{(a+bu)^2} = \frac{1}{b^2}\left(\ln|a+bu| + \frac{a}{a+bu}\right) + C$

$$\int \frac{30x^5}{(4+5x^3)^2}\,dx = 10\left[\frac{1}{25}\left(\ln\left|4+5x^3\right| + \frac{4}{4+5x^3}\right)\right] + C = \frac{2}{5}\left(\ln\left|4+5x^3\right| + \frac{4}{4+5x^3}\right) + C$$

$$\int \frac{30x^5}{(4+5x^3)^2}\,dx = \frac{2}{5}\ \ln\left|4+5x^3\right| + \frac{8}{5(4+5x^3)} + C$$

Example 2:

Use the table to determine $\displaystyle\int \frac{8}{x(2+3x^4)}\,dx = \int \frac{1}{x^4(2+3x^4)}\,8x^3\,dx$

substitute: $u = x^4\,,\ du = 4x^3\,dx\,,\ 2\,du = 8x^3\,dx\,,\ a = 2\,,\ b = 3 \implies \displaystyle\int \frac{1}{u(a+bu)}\,2\,du \implies$ 2 (answer F2)

Use F2: $\displaystyle\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$

$$\int \frac{8}{x(2+3x^4)}\,dx = 2\left[\frac{1}{2}\ln\left|\frac{x^4}{2+3x^4}\right|\right] + C = \ln\left|\frac{x^4}{2+3x^4}\right| + C$$

Example 3:

Use the table to determine $\displaystyle\int \frac{x^3}{\sqrt{3+4x^2}}\,dx = \int \frac{x^2}{\sqrt{3+4x^2}}\,x\,dx$

substitute: $u = x^2\,,\ du = 2x\,dx\,,\ \frac{1}{2}\,du = x\,dx\,,\ a = 3\,,\ b = 4 \implies \displaystyle\int \frac{u}{\sqrt{a+bu}}\,\frac{1}{2}\,du \implies \frac{1}{2}$ (answer F15)

Use F15: $\displaystyle\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu} + C$

$$\int \frac{x^3}{\sqrt{3+4x^2}}\,dx = \frac{1}{2}\left[\frac{2}{3(16)}(4x^2-6)\sqrt{3+4x^2}\right] + C = \frac{1}{24}\,(2x^2-3)\sqrt{3+4x^2} + C$$

Example 4:

Use the table to determine $\int \frac{4x-3}{\sqrt{2+3x}}\,dx$ rewrite $4\int \frac{x\,dx}{\sqrt{2+3x}} - 3\int \frac{dx}{\sqrt{2+3x}}$

substitute: $u = x$, $du = dx$, $a = 2$, $b = 3$

$$4\int \frac{x\,dx}{\sqrt{2+3x}} \implies 4\int \frac{u\,du}{\sqrt{a+bu}} \implies \text{(answer 4F15)} \implies \text{Use F15: } \int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu} + C$$

substitution: $u = 2+3x$, $du = 3dx$, $\frac{1}{3}\,du = dx$

$$-3\int \frac{dx}{\sqrt{2+3x}} \implies -3\int \frac{\frac{1}{3}\,du}{u^{1/2}} \implies \int -u^{-1/2}\,du = -2u^{1/2} + C$$

therefore $4\int \frac{x\,dx}{\sqrt{2+3x}} - 3\int \frac{dx}{\sqrt{2+3x}} = 4\left[\frac{2}{3(9)}(3x-4)\sqrt{2+3x}\right] - 2(2+3x)^{1/2} + C$

$$\int \frac{4x-3}{\sqrt{2+3x}}\,dx = \frac{8}{27}(3x-4)\sqrt{2+3x} - 2\sqrt{2+3x} + C = \frac{2}{27}\sqrt{2+3x}\,(12x-43) + C$$

Example 5:

Use the table to determine $\int \frac{(x-3)^2}{\sqrt{x^2-6x+5}}\,dx$

complete the square method: coefficient of x^2 must be 1 ; then add & subtract $\left(\frac{\text{coefficient of } x}{2}\right)^2$

in this problem : $(x^2-6x)+5 = (x^2-6x+9)+5-9 = (x-3)^2 - 4 \rightarrow \int \frac{(x-3)^2\,dx}{\sqrt{(x-3)^2-4}}$

substitute: $u = x-3$, $du = dx$, $a^2 = 4$, $a = 2 \implies \int \frac{u^2\,du}{\sqrt{u^2-a^2}} \implies$ (answer F28)

Use F28: $\int \frac{u^2\,du}{\sqrt{u^2 \pm a^2}} = \frac{1}{2}\left(u\sqrt{u^2 \pm a^2} \mp a^2 \ln\left|u + \sqrt{u^2 \pm a^2}\right|\right) + C$

$$\int \frac{(x-3)^2}{\sqrt{x^2-6x+5}}\,dx = \frac{1}{2}\left((x-3)\sqrt{x^2-6x+5} + 4\ln\left|x-3+\sqrt{x^2-6x+5}\right|\right) + C$$

$$\int \frac{(x-3)^2}{\sqrt{x^2-6x+5}}\,dx = \frac{1}{2}(x-3)\sqrt{x^2-6x+5} + 2\ln\left|x-3+\sqrt{x^2-6x+5}\right| + C$$

Example 6:

Use the table to determine $\displaystyle\int \frac{\sqrt{5+4x-x^2}}{x-2}\,dx$

complete the square method: coefficient of x^2 must be 1 ; then add & subtract $\left(\frac{\text{coefficient of } x}{2}\right)^2$

in this problem : $5-(-4x+x^2) = 5+4-(x^2-4x+4) = 9-(x-2)^2 \rightarrow \displaystyle\int \frac{\sqrt{9-(x-2)^2}\,dx}{x-2}$

substitute: $u = x-2\,,\ du = dx\,,\ a^2 = 9\,,\ a = 3 \implies \displaystyle\int \frac{\sqrt{a^2-u^2}\,du}{u} \implies$ (answer F21)

Use F21: $\displaystyle\int \frac{\sqrt{a^2-u^2}\,du}{u} = \sqrt{a^2-u^2} - a\,\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$

$$\int \frac{\sqrt{5+4x-x^2}}{x-2}\,dx = \sqrt{5+4x-x^2} - 3\,\ln\left|\frac{3+\sqrt{5+4x-x^2}}{x-2}\right| + C$$

Example 7:

Use the table to determine $\displaystyle\int \frac{x^2-1}{\sqrt{x^2+4}}\,dx$ rewrite $\displaystyle\int \frac{x^2\,dx}{\sqrt{x^2+4}} - \int \frac{dx}{\sqrt{x^2+4}}$

substitute: $u^2 = x^2\,,\ u = x\,,\ du = dx\,,\ a^2 = 4 \implies \displaystyle\int \frac{x^2\,dx}{\sqrt{x^2+4}} \implies \int \frac{u^2\,du}{\sqrt{u^2+a^2}} \implies$ (answer F28)

Use F28: $\displaystyle\int \frac{u^2\,du}{\sqrt{u^2\pm a^2}} = \frac{1}{2}\left(u\,\sqrt{u^2\pm a^2} \mp a^2\,\ln\left|u+\sqrt{u^2\pm a^2}\right|\right) + C$

substitute: $u^2 = x^2\,,\ u = x\,,\ du = dx\,,\ a^2 = 4 \implies -\displaystyle\int \frac{dx}{\sqrt{x^2+4}} \implies -\int \frac{du}{\sqrt{u^2+a^2}} \implies$ (answer – F26)

Use F26: $\displaystyle\int \frac{du}{\sqrt{u^2\pm a^2}} = \ln\left|u+\sqrt{u^2\pm a^2}\right| + C$

$$\int \frac{x^2\,dx}{\sqrt{x^2+4}} - \int \frac{dx}{\sqrt{x^2+4}} = \frac{1}{2}\left(x\,\sqrt{x^2+4} - 4\,\ln\left|x+\sqrt{x^2+4}\right|\right) - \ln\left|x+\sqrt{x^2+4}\right| + C$$

$$\int \frac{x^2-1}{\sqrt{x^2+4}}\,dx = \frac{1}{2}\,x\,\sqrt{x^2+4} - 3\,\ln\left|x+\sqrt{x^2+4}\right| + C$$

Example 8:

Use the table to determine $\displaystyle\int \frac{4x\,dx}{\sqrt{(2+x^2)(5+x^2)}} = \int 2\frac{2x\,dx}{\sqrt{(2+x^2)(5+x^2)}}$

substitute: $u = x^2$, $du = 2x\,dx$, $a = 2$, $b = 5 \implies \displaystyle\int 2\frac{du}{\sqrt{(a+u)(b+u)}} \implies$ 2 (answer F39)

Use F39: $\displaystyle\int \frac{du}{\sqrt{(a+u)(b+u)}} = \ln\left|\frac{a+b}{2} + u + \sqrt{(a+u)(b+u)}\right| + C$

$$\int \frac{4x\,dx}{\sqrt{(2+x^2)(5+x^2)}} = 2\,\ln\left|\frac{2+5}{2} + x^2 + \sqrt{(2+x^2)(5+x^2)}\right| + C = 2\,\ln\left|\frac{7}{2} + x^2 + \sqrt{(2+x^2)(5+x^2)}\right| + C$$

Example 9:

Use the table to determine $\displaystyle\int 6x\sqrt{\frac{4+x^2}{9+x^2}}\,dx = \int 3\sqrt{\frac{4+x^2}{9+x^2}}\,2x\,dx$

substitute: $u = x^2$, $du = 2x\,dx$, $a = 4$, $b = 9 \implies \displaystyle\int 3\sqrt{\frac{a+u}{b+u}}\,du \implies$ 3 (answer F38)

Use F38: $\displaystyle\int \sqrt{\frac{a+u}{b+u}}\,du = \sqrt{(a+u)(b+u)} + (a-b)\,\ln\left(\sqrt{a+u} + \sqrt{b+u}\right) + C$

$$\int 6x\sqrt{\frac{4+x^2}{9+x^2}}\,dx = 3\left[\sqrt{(4+x^2)(9+x^2)} + (4-9)\,\ln\left(\sqrt{4+x^2} + \sqrt{9+x^2}\right)\right] + C$$

$$\int 6x\sqrt{\frac{4+x^2}{9+x^2}}\,dx = 3\sqrt{(4+x^2)(9+x^2)} - 15\,\ln\left(\sqrt{4+x^2} + \sqrt{9+x^2}\right) + C$$

Example 10:

Use the table to determine $\displaystyle\int \frac{8x}{7-3\,e^{2x^2}}\,dx = \int 4\,\frac{2x\,dx}{7-3\,e^{2x^2}}$

substitute: $u = x^2\,,\ du = 2x\,dx\,,\ a = 7\,,\ b = -3\,,\ c = 2 \implies \displaystyle\int 4\,\frac{du}{a+be^{cu}} \implies 4$ (answer F33)

Use F33: $\displaystyle\int \frac{du}{a+be^{cu}} = \frac{1}{ac}\left(cu - \ln\left|a+be^{cu}\right|\right) + C$

$$\int \frac{8x}{7-3\,e^{2x^2}}\,dx = 4\left[\frac{1}{(7)(2)}\left(2x^2 - \ln\left|7-3e^{2x^2}\right|\right)\right] + C$$

$$\int \frac{8x}{7-3\,e^{2x^2}}\,dx = \frac{4}{7}\,x^2 - \frac{2}{7}\,\ln\left|7-3e^{2x^2}\right| + C$$

Example 11:

Use the table to determine $\displaystyle\int \frac{4}{\sqrt{x^2-6x+10}}\,dx$

complete the square method: coefficient of x^2 must be 1 ; then add & subtract $\left(\dfrac{\text{coefficient of } x}{2}\right)^2$

in this problem : $(x^2-6x)+10 = (x^2-6x+9)+10-9 = (x-3)^2+1 \rightarrow \displaystyle\int \frac{4\,dx}{\sqrt{(x-3)^2+1}}$

substitute: $u = x-3\,,\ du = dx\,,\ a^2 = 1\,,\ a = 1 \implies \displaystyle\int \frac{4\,du}{\sqrt{u^2-a^2}} \implies 4$ (answer F26)

Use F26: $\displaystyle\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

$$\int \frac{4}{\sqrt{x^2-6x+10}}\,dx = 4\,\ln\left|x-3+\sqrt{x^2-6x+10}\right| + C$$

Example 12:

Use the table to determine $\displaystyle\int \frac{6}{(x+2)\sqrt{x^2+4x+8}}\,dx$

complete the square method: coefficient of x^2 must be 1 ; then add & subtract $\left(\frac{\text{coefficient of } x}{2}\right)^2$

in this problem : $(x^2+4x)+8 = (x^2+4x+4)+8-4 = (x+2)^2+4 \rightarrow \displaystyle\int \frac{6}{(x+2)\sqrt{(x+2)^2+4}}$

substitute: $u = x+2\,,\ du = dx\,,\ a^2 = 4\,,\ a = 2 \implies \displaystyle\int 6\,\frac{du}{u\sqrt{u^2+a^2}} \implies$ 6 (answer F27)

Use F27: $\displaystyle\int \frac{du}{u\sqrt{u^2+a^2}} = \frac{1}{a}\ln\left|\frac{\sqrt{u^2+a^2}-a}{u}\right| + C$

$$\int \frac{6}{(x+2)\sqrt{x^2+4x+8}}\,dx = 6\left[\frac{1}{2}\ln\left|\frac{\sqrt{x^2+4x+8}-2}{x+2}\right|\right] + C$$

$$\int \frac{6}{(x+2)\sqrt{x^2+4x+8}}\,dx = 3\ln\left|\frac{\sqrt{x^2+4x+8}-2}{x+2}\right| + C$$

Example 13:

Use the table to determine $\displaystyle\int 6x^2\ln^2 x\,dx$

must use F31: $\displaystyle\int u^n \ln^m u\,du = \frac{u^{n+1}}{n+1}\ln^m u - \frac{m}{n+1}\int u^n \ln^{m-1} u\,du$

substitute: $u = x\,,\ du = dx\,,\ n = 2\,,\ m = 2 \implies \displaystyle\int 6\,u^n \ln^m u\,du \implies$ 6 (answer F31)

$$\int 6x^2\ln^2 x\,dx = 6\left[\frac{x^3}{3}\ln^2 x - \frac{2}{3}\int x^2\ln x\,dx\right] = 2x^3\ln^2 x - 4\int x^2\ln x\,dx$$

use F31 with $n = 2\,,\ n = 1 \implies \displaystyle\int x^2\ln x\,dx = \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2\,dx = \frac{x^3}{3}\ln x - \frac{1}{3}\frac{x^3}{3}$

$$\int 6x^2\ln^2 x\,dx = 2x^3\ln^2 x - 4\left[\frac{x^3}{3}\ln x - \frac{1}{3}\frac{x^3}{3}\right] + C = 2x^3\ln^2 x - \frac{4}{3}x^3\ln x + \frac{4}{9}x^3 + C$$