Trapezoidal Rule

The Trapezoidal Rule is to approximate $\int_a^b f(x)\,dx$

The formula used in class is as follows:

Use trapezoidal Rule with n (positive number) to estimate $\int_a^b f(x)\,dx$

Let $h = \dfrac{b-a}{n}$ and

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[\,f(a) + 2\,f(a+h) + 2\,f(a+2h) + 2\,f(a+3h) + \cdots + f(b)\,\right]$$

Example 1:

Use the Trapezoidal Rule with $n = 6$ to estimate $\int_0^3 e^{x^2-4}\,dx$. Give answer with 3 decimals places.

h=$\dfrac{b-a}{n} = \dfrac{3-0}{6} = 0.5$

$$\int_0^3 e^{x^2-4}\,dx \approx \frac{h}{2}\,[\text{total}]$$

$$\approx \frac{0.5}{2}\,[169.9011083] \approx 42.475$$

x	$f(x) = e^{x^2-4}$
0	$f(0) = 0.01832$
0.5	$2f(0.5) = 2(0.023517745) = 0.047035491$
1	$2f(1) = 2(0.049787068) = 0.099574136$
1.5	$2f(1.5) = 2(0.173773943) = 0.347547886$
2	$2f(2) = 2(1) = 2$
2.5	$2f(2.5) = 2(9.487735836) = 18.97547167$
3	$f(3) = 148.4131591$
Total =	169.9011083

Example 2:

Use the Trapezoidal Rule with $n = 4$ to estimate $\int_{-1}^3 \dfrac{1}{\ln(x+4)}\,dx$. Give answer with 3 decimals places.

h=$\dfrac{b-a}{n} = \dfrac{3-(-1)}{4} = 1$

$$\int_{-1}^3 \frac{1}{\ln(x+4)}\,dx \approx \frac{h}{2}\,[\text{total}]$$

$$\approx \frac{1}{2}\,[5.225723731] \approx 2.613$$

x	$f(x) = \dfrac{1}{\ln(x+4)}$
-1	$f(-1) = 0.910239226$
0	$2f(0) = 2(0.72134752) = 1.442695041$
1	$2f(1) = 2(0.621334934) = 1.242669869$
2	$2f(2) = 2(0.558110626) = 1.116221253$
3	$f(3) = 0.513898342$
Total =	5.225723731

Example 3:

Use the Trapezoidal Rule with $n = 4$ to estimate $\int_0^4 \frac{10}{\sqrt{x^3+8}}\,dx$. Give answer with 3 decimals places.

h$=\frac{b-a}{n} = \frac{4-0}{4} = 1$

$\int_0^4 \frac{10}{\sqrt{x^3+8}}\,dx \approx \frac{h}{2}\,[\text{total}]$

$\approx \frac{1}{2}\,[19.76132889] \approx 9.881$

x	$f(x) = \frac{10}{\sqrt{x^3+8}}$
0	$f(0) = 3.535533906$
1	$2f(1) = 2(3.333333333) = 6.666666667$
2	$2f(2) = 2(2.5) = 5$
3	$2f(3) = 2(1.690308509) = 3.380617019$
4	$f(4) = 1.178511302$
Total =	19.76132889

Simpson's Rule

The Simpson's Rule is to approximate $\int_a^b f(x)\,dx$

The formula used in class is as follows:

Use Simpson's Rule with n (positive even number) to estimate $\int_a^b f(x)\,dx$

Let $h = \frac{b-a}{n}$ and

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[\, f(a) + 4\,f(a+h) + 2\,f(a+2h) + 4\,f(a+3h) + \cdots + f(b)\,\right]$$

Example 4:

Use the Simpson's Rule with $n = 6$ to estimate $\int_0^3 e^{x^2-4}\,dx$. Give answer with 3 decimals places.

h$=\frac{b-a}{n} = \frac{3-0}{6} = 0.5$

$\int_0^3 e^{x^2-4}\,dx \approx \frac{h}{3}\,[\text{total}]$

$\approx \frac{0.5}{3}\,[189.2711633] \approx 31.545$

x	$f(x) = e^{x^2-4}$
0	$f(0) = 0.01832$
0.5	$4f(0.5) = 4(0.023517745) = 0.094070983$
1	$2f(1) = 2(0.049787068) = 0.099574136$
1.5	$4f(1.5) = 4(0.173773943) = 0.695095773$
2	$2f(2) = 2(1) = 2$
2.5	$4f(2.5) = 4(9.487735836) = 37.95094335$
3	$f(3) = 148.4131591$
Total =	189.2711633

Example 5:

Use the Simpson's Rule with $n = 4$ to estimate $\int_{-1}^{3} \frac{1}{\ln(x+4)}\, dx$. Give answer with 3 decimals places.

h=$\frac{b-a}{n} = \frac{3-(-1)}{4} = 1$

$\int_{-1}^{3} \frac{1}{\ln(x+4)}\, dx \approx \frac{h}{3}[\text{total}]$

$\approx \frac{1}{3}[7.784640025] \approx 2.595$

x	$f(x) = \frac{1}{\ln(x+4)}$
-1	$f(-1) = 0.910239226$
0	$4f(0) = 4(0.72134752) = 2.885390082$
1	$2f(1) = 2(0.621334934) = 1.242669869$
2	$4f(2) = 4(0.558110626) = 2.232442506$
3	$f(3) = 0.513898342$
Total =	7.784640025

Example 6:

Use the Simpson's Rule with $n = 4$ to estimate $\int_{0}^{4} \frac{10}{\sqrt{x^3+8}}\, dx$. Give answer with 3 decimals places.

h=$\frac{b-a}{n} = \frac{4-0}{4} = 1$

$\int_{0}^{4} \frac{10}{\sqrt{x^3+8}}\, dx \approx \frac{h}{3}[\text{total}]$

$\approx \frac{1}{3}[29.80861258] \approx 9.936$

x	$f(x) = \frac{10}{\sqrt{x^3+8}}$
0	$f(0) = 3.535533906$
1	$4f(1) = 4(3.333333333) = 13.33333333$
2	$2f(2) = 2(2.5) = 5$
3	$4f(3) = 4(1.690308509) = 6.761234038$
4	$f(4) = 1.178511302$
Total =	29.80861258

Trapezoidal Rule versus Simpson's Rule

Estimate $\int_{0}^{3} e^{x^2-4}\, dx \approx 26.458$

trapezoidal rule in example 1 gives 42.475

Simpson's rule in example 4 gives 31.545

Estimate $\int_{-1}^{3} \frac{1}{\ln(x+4)}\, dx \approx 2.593$

trapezoidal rule in example 2 gives 2.613

Simpson's rule in example 5 gives 2.595

Estimate $\int_{0}^{4} \frac{10}{\sqrt{x^3+8}}\, dx \approx 9.915$

trapezoidal rule in example 3 gives 9.881

Simpson's rule in example 6 gives 9.936

Note:

These examples show that the estimate is better with Simpson's Rule for same n (even positive integer).